Day 3

- Gravitational-wave concepts (with Dr. Jocelyn Read)
- Special guest: Haroon Khan (NASA)
- Choose one head-on collision on binary black holes and start the calculation

Two kinds of time travel

- Travel from the present to…

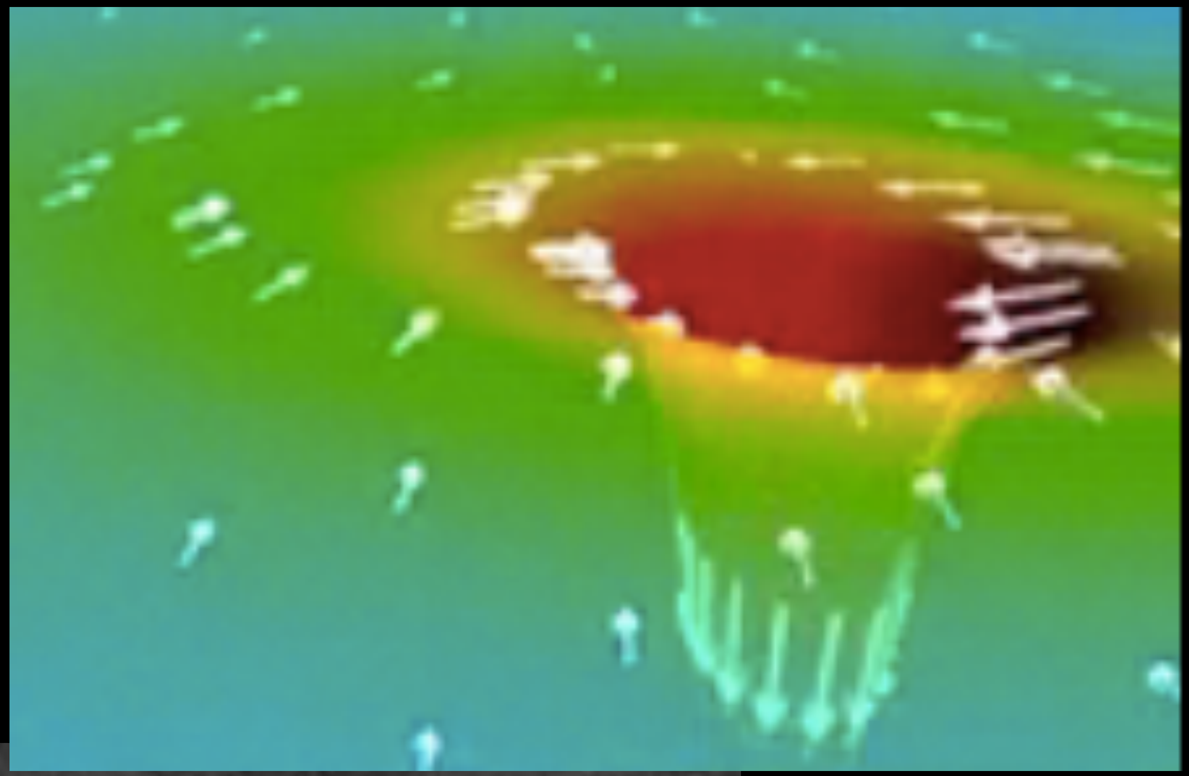
The future

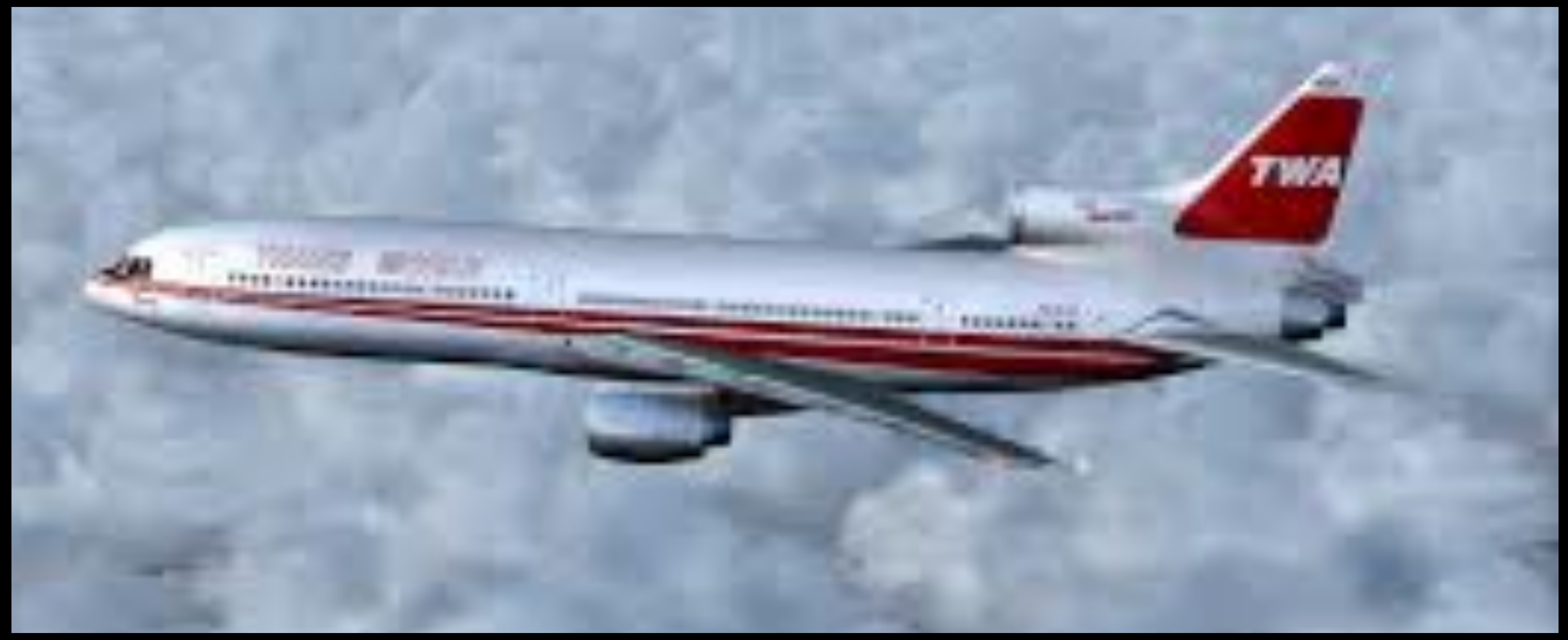
The past

Images courtesy backtothefuture.wikia.com, oocities, “Back to the Future”

Forward time travel

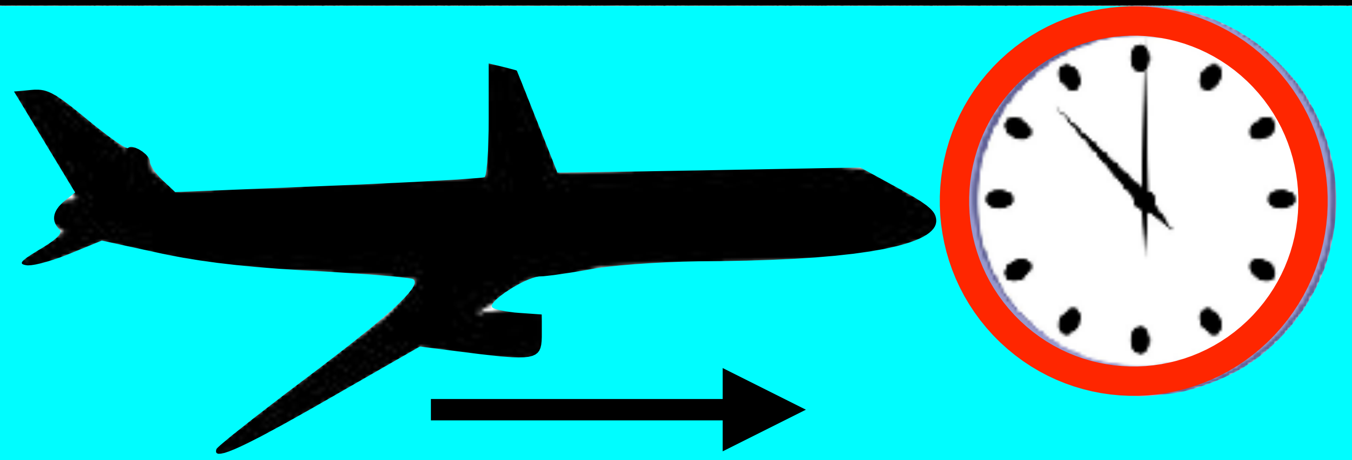
- Make your time flow slower
  - –Move closer to massive object
  - –Move faster
- Hafele & Keating 1971
  - –Fly plane clock around world
  - –Compare with ground clock before, after flight





Images courtesy SXS Collaboration, wikipedia

Forward time travel in 1971



Theory of relativity predicts…

Farther from Earth: ticks faster vs. ground

144±14 ns *more* than ground

Moves faster: ticks slower vs. ground

184±18 ns *less* than ground

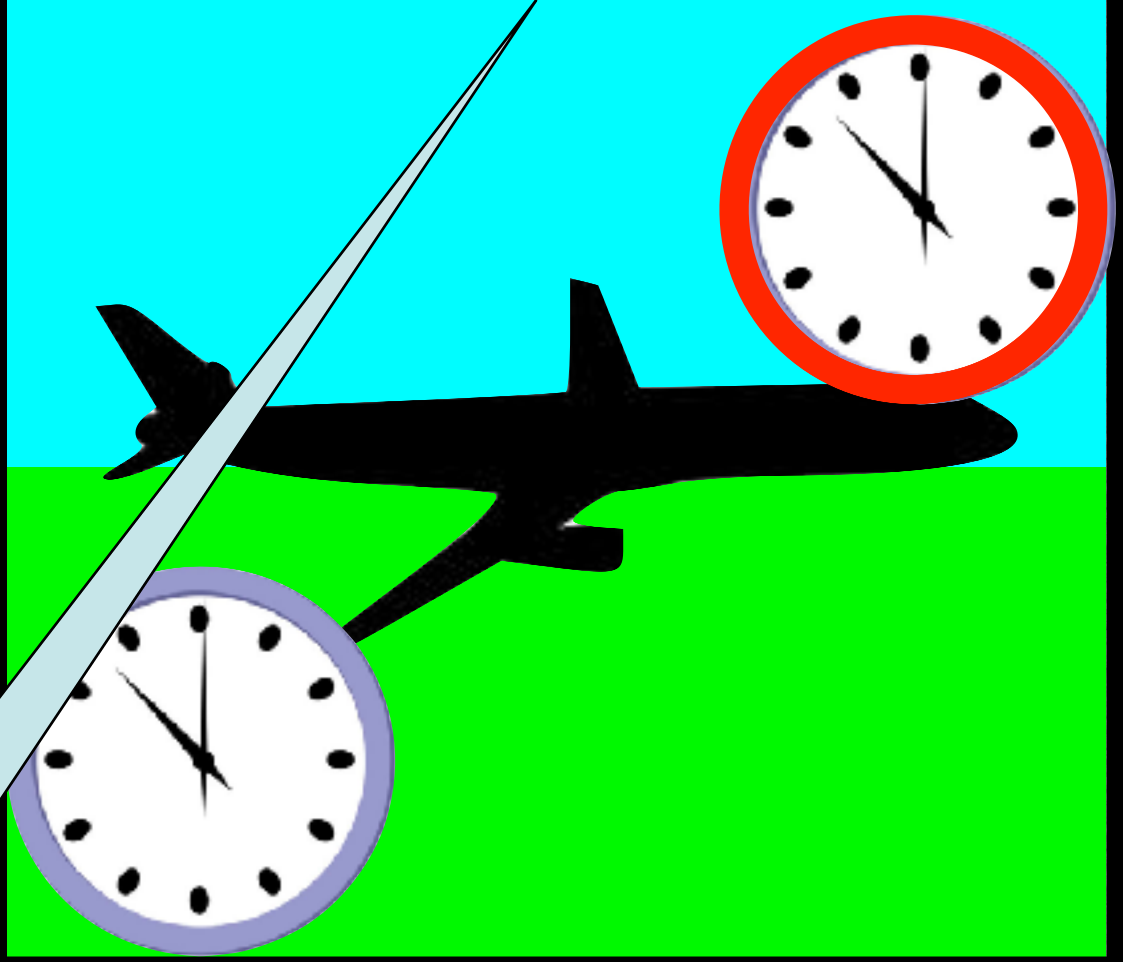
Bottome line:

40±23 ns *less* than ground

1 ns = 1 billionth of a second (ns)

Result: compare clocks after flight around the world

59±10 ns less than ground

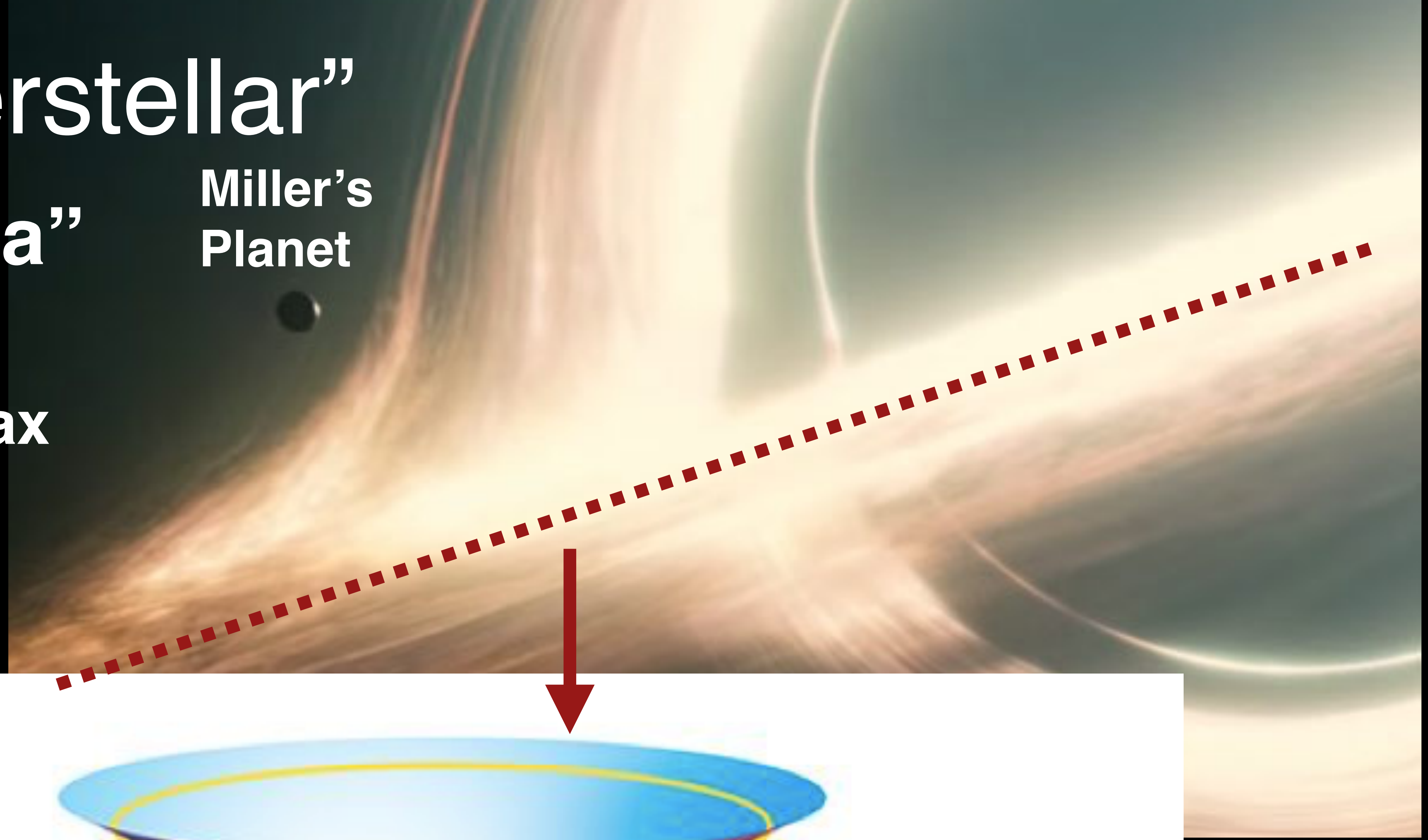


Clock & passengers went 59±10 billionths of a second into the future!

Time travel in “Interstellar”

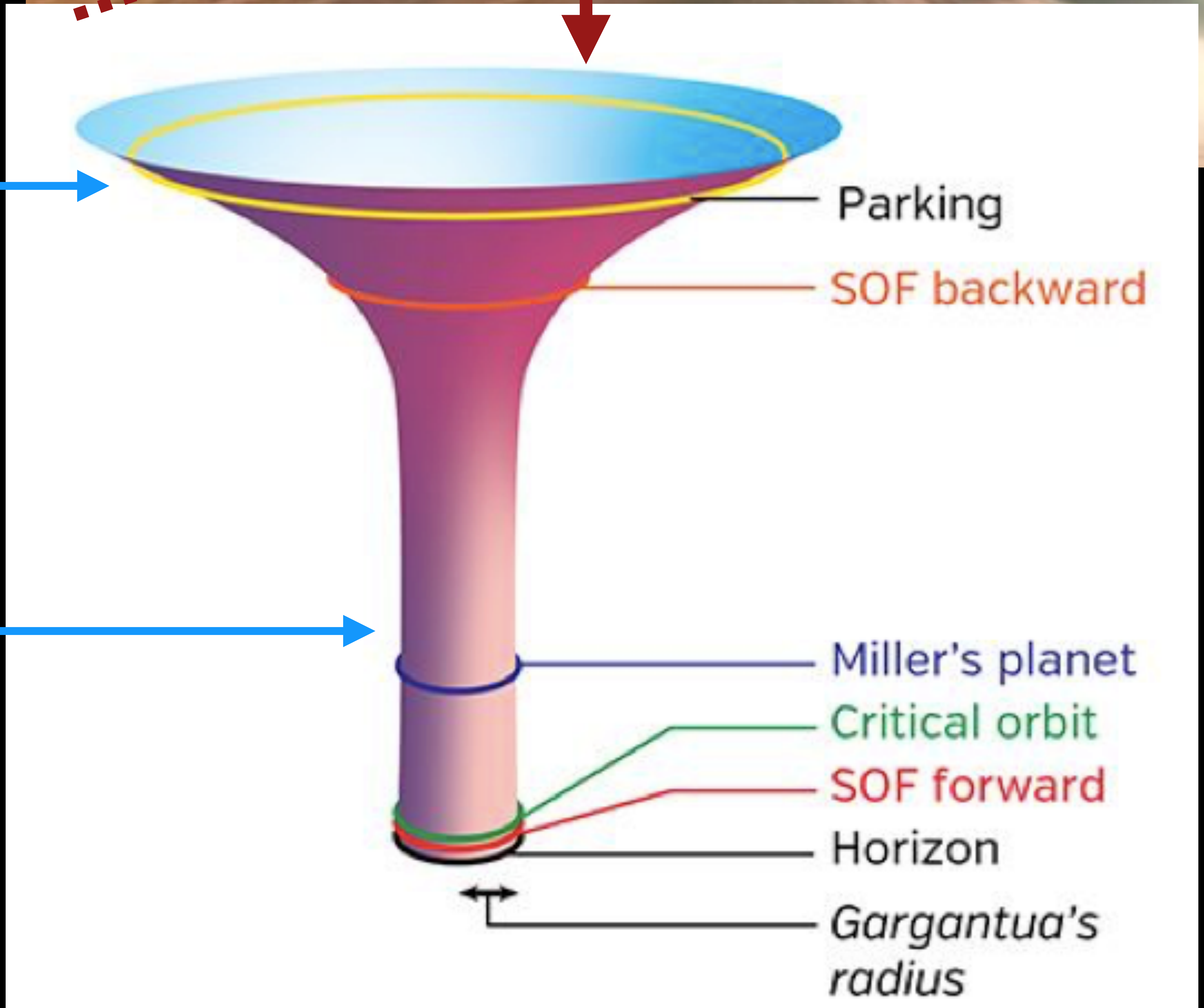
Black hole “Gargantua”

Mass: 100 000 000 ☀
Spin: 99.999999999999% max



7 years

1 hour



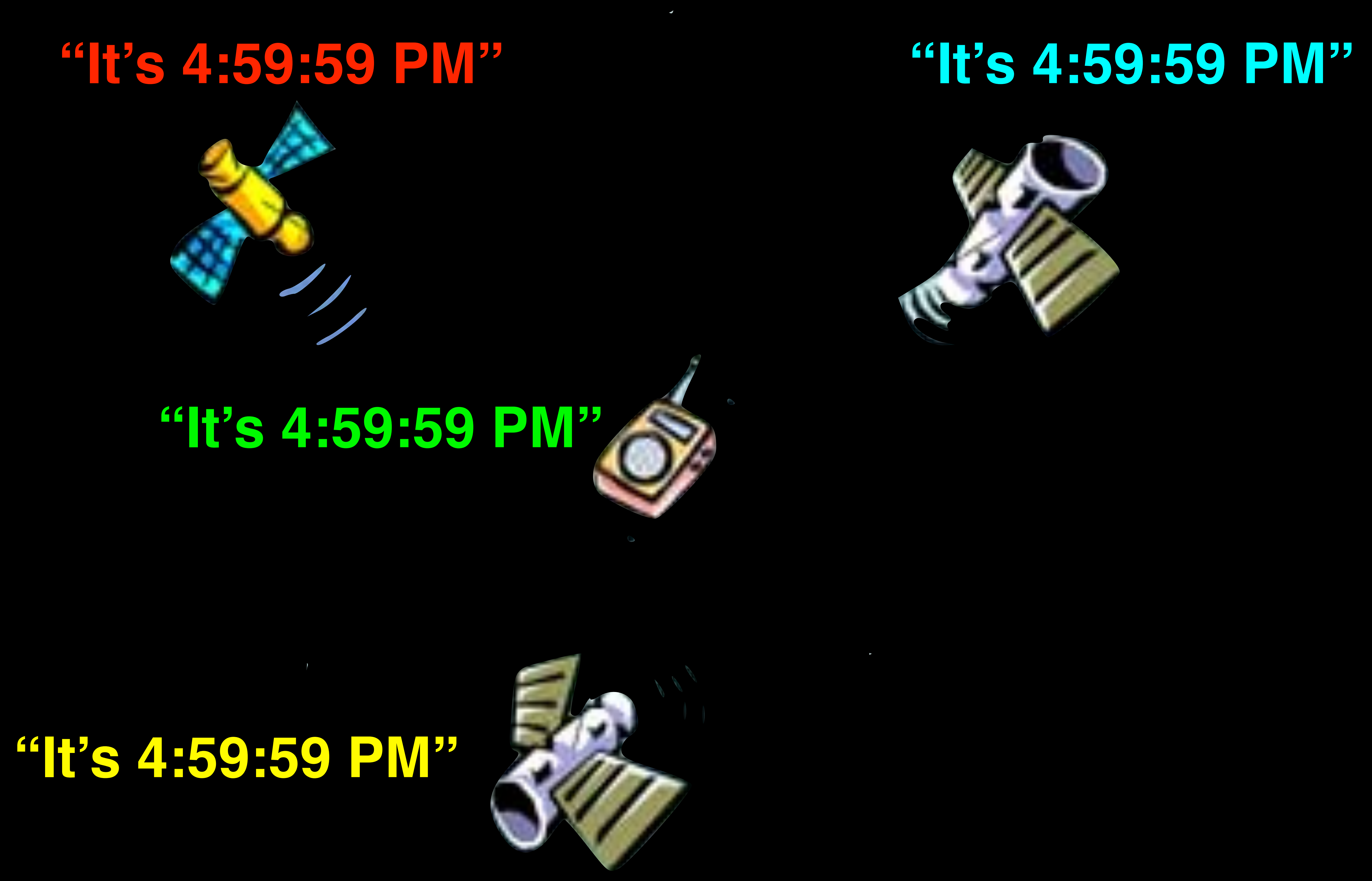
Images courtesy Kip Thorne, Paramount

GPS

- How does GPS work?

GPS

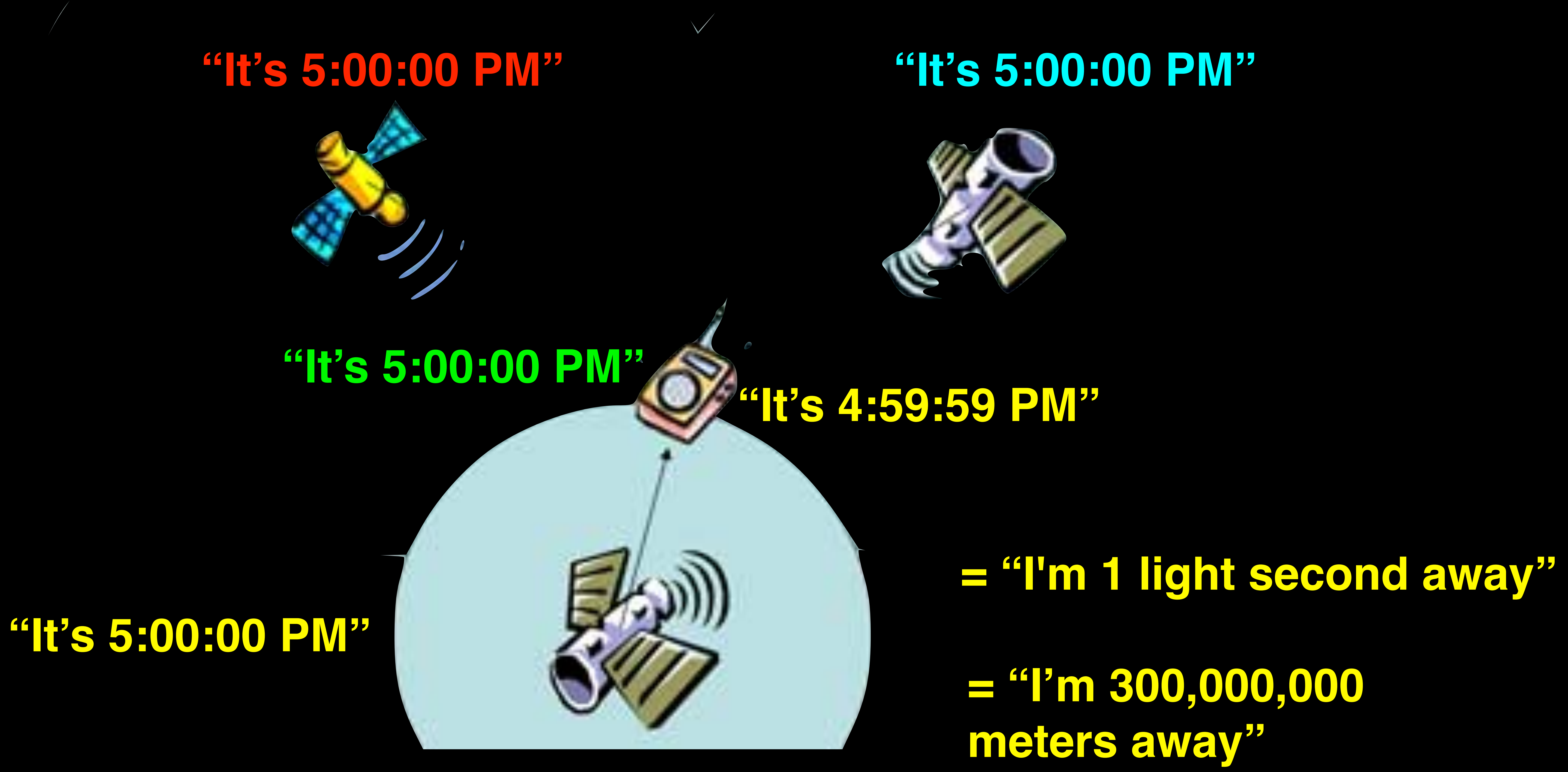
- How does GPS work?



Images courtesy NOAA, how-gps-works.com

GPS

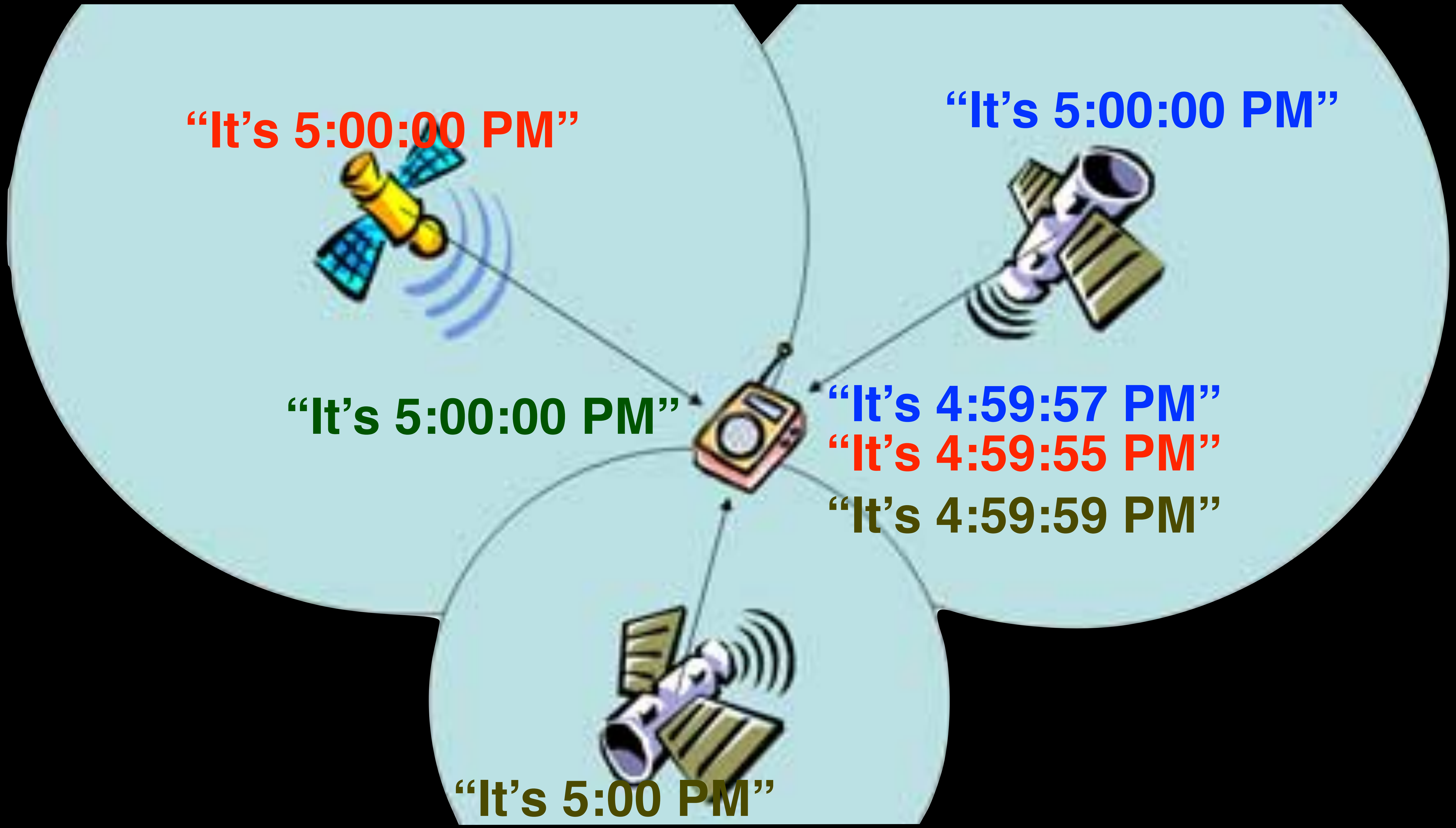
- How does GPS work?



Images courtesy NOAA, how-gps-works.com

GPS

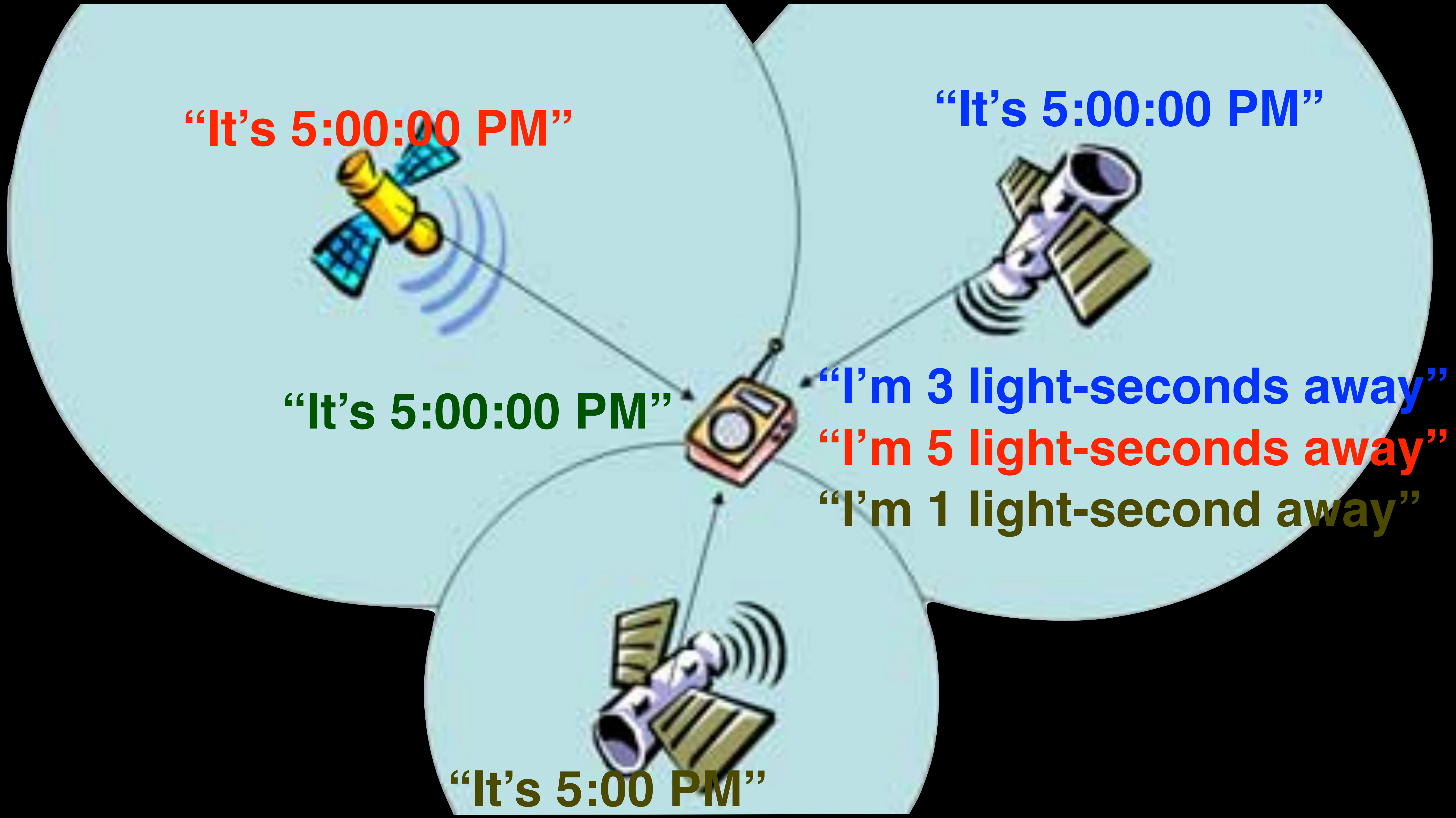
- How does GPS work?



Images courtesy NOAA, how-gps-works.com

GPS

- How does GPS work?



Images courtesy NOAA, how-gps-works.com

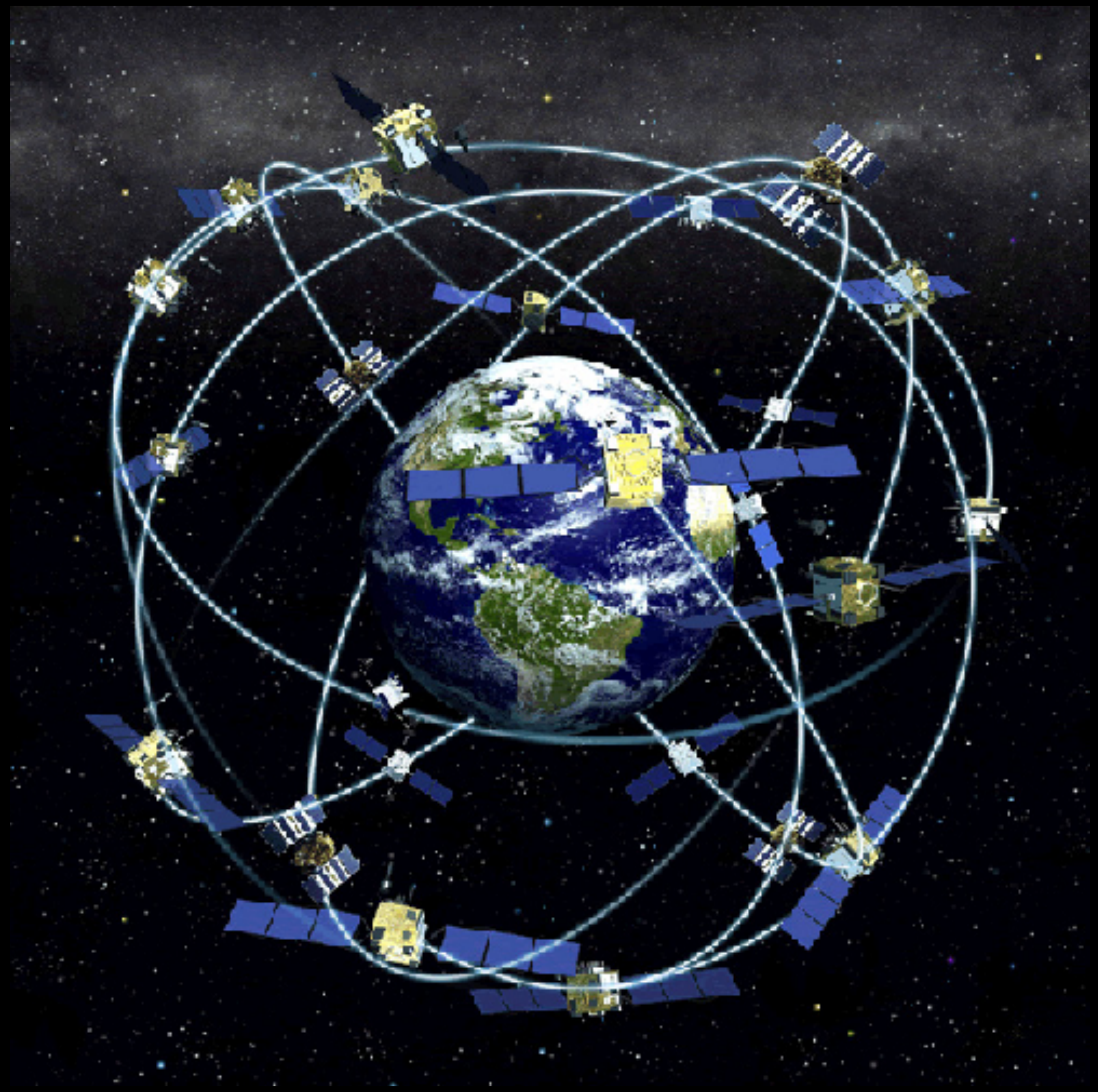
GPS

- How does GPS work?

Images courtesy NOAA, how-gps-works.com

GPS & forward time travel

- GPS must account for both "time travel" effects



Goal: position accuracy of about 2 m

Light travels 2 m in about 7 ns

So clocks really give time to ns precision:
"It's 4:59:59.123456789 PM"

That's no problem for atomic clocks, but…

Satellite clocks are higher, moving: tick differently!

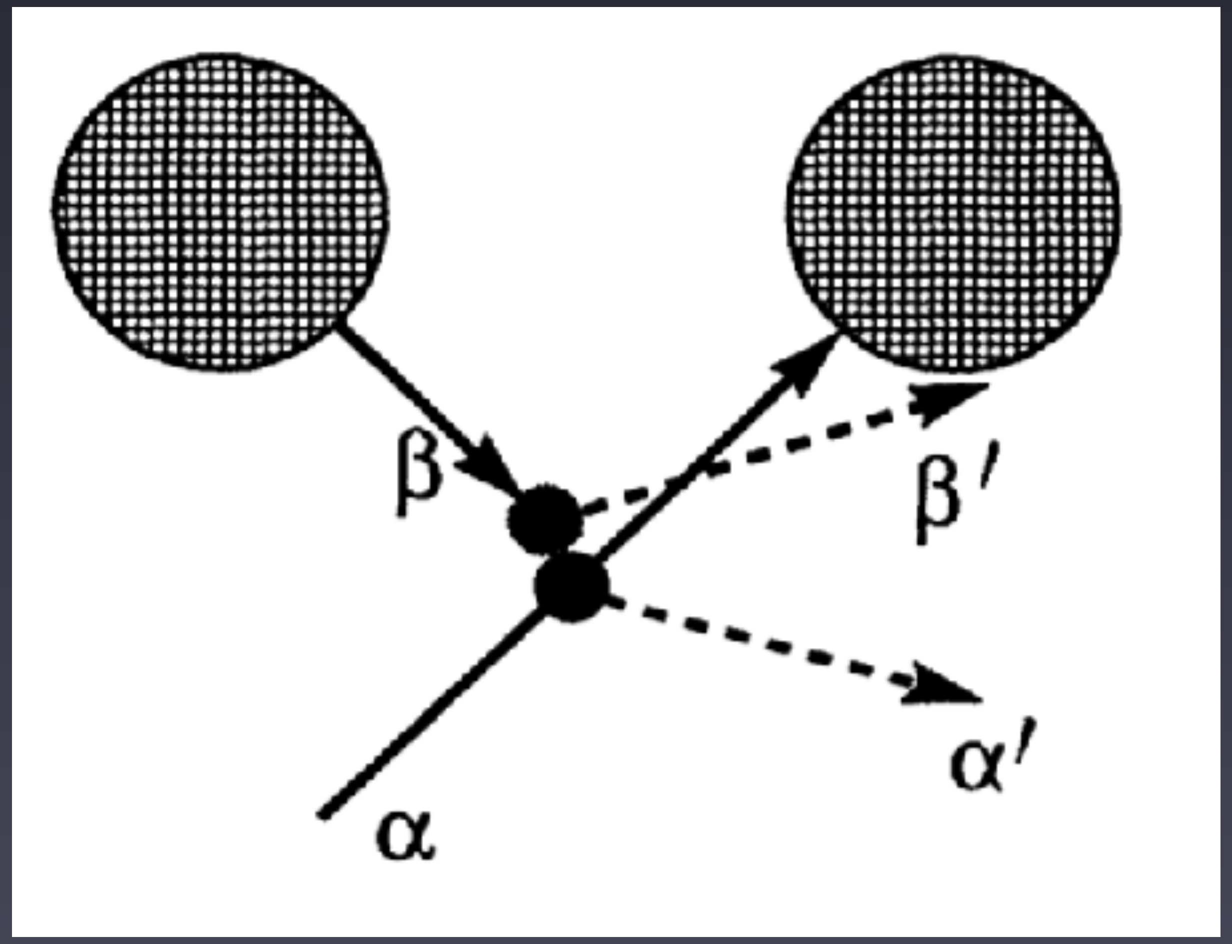
Ignore this, and errors start to build up, exceeding 2 m in less than a minute

Images courtesy NOAA, how-gps-works.com

Backward time travel

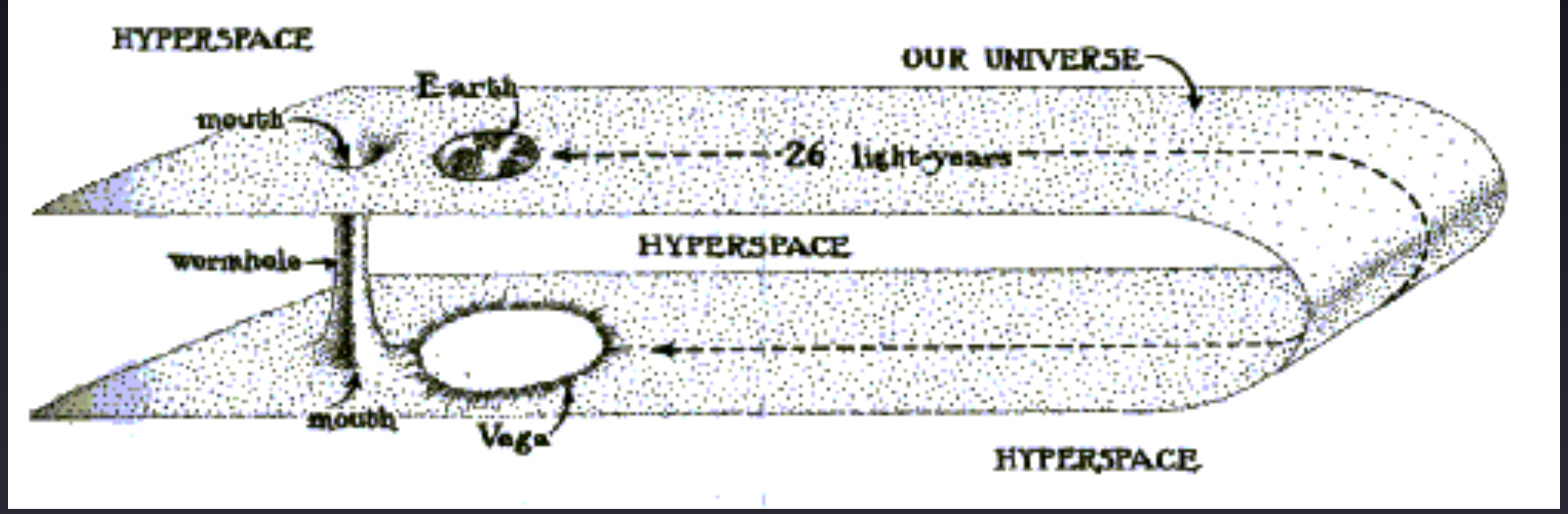
- "Matricide paradox"
  - Go back in time and prevent yourself from being born?
- "Billiard ball paradox" (Polchinski, 1988)
  - Can ball go back in time & collide with itself, preventing itself from going back in time?
  - Echeverria, Klinkhammer, and Thorne [EKT], 1991

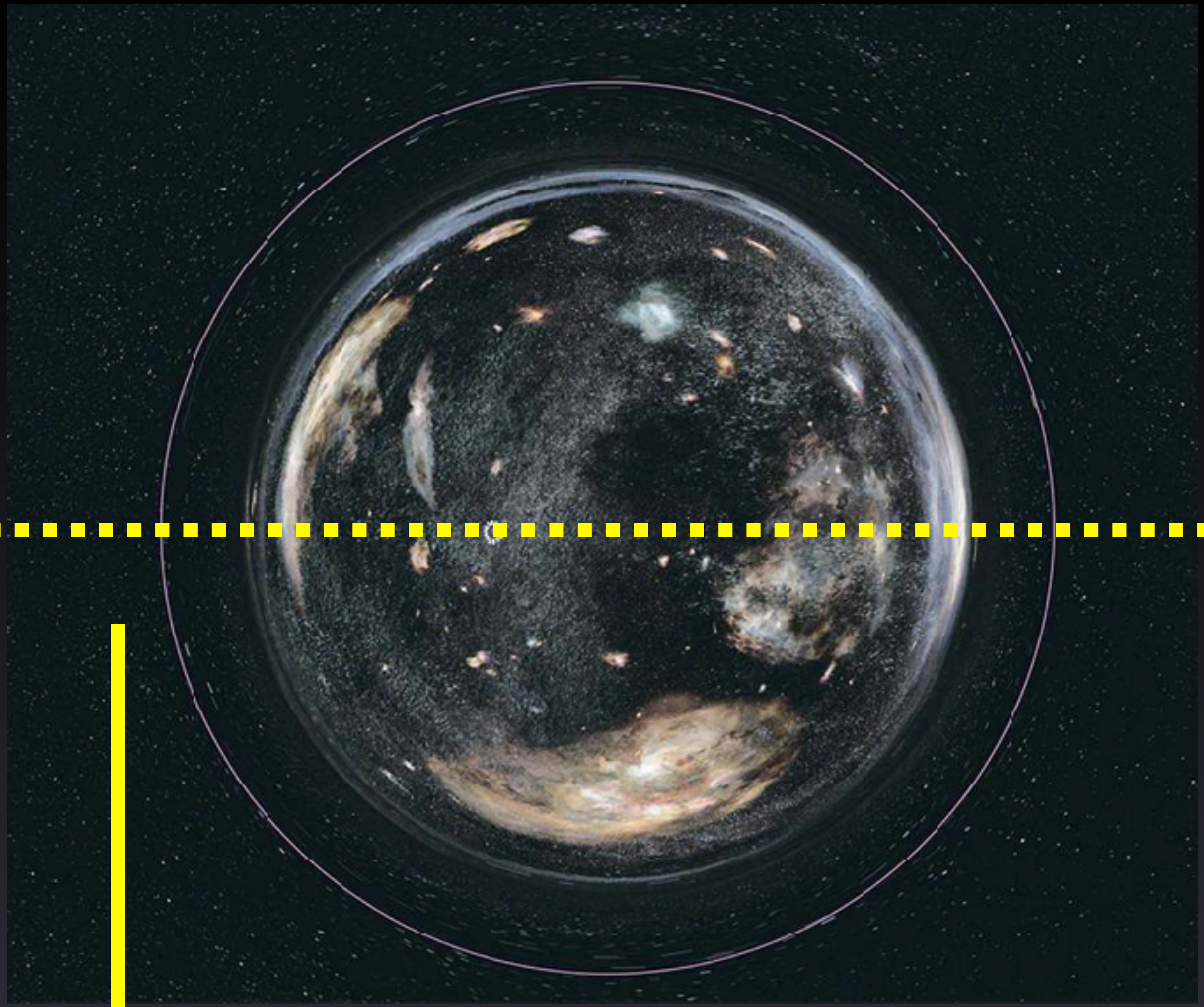
Image courtesy wikipedia



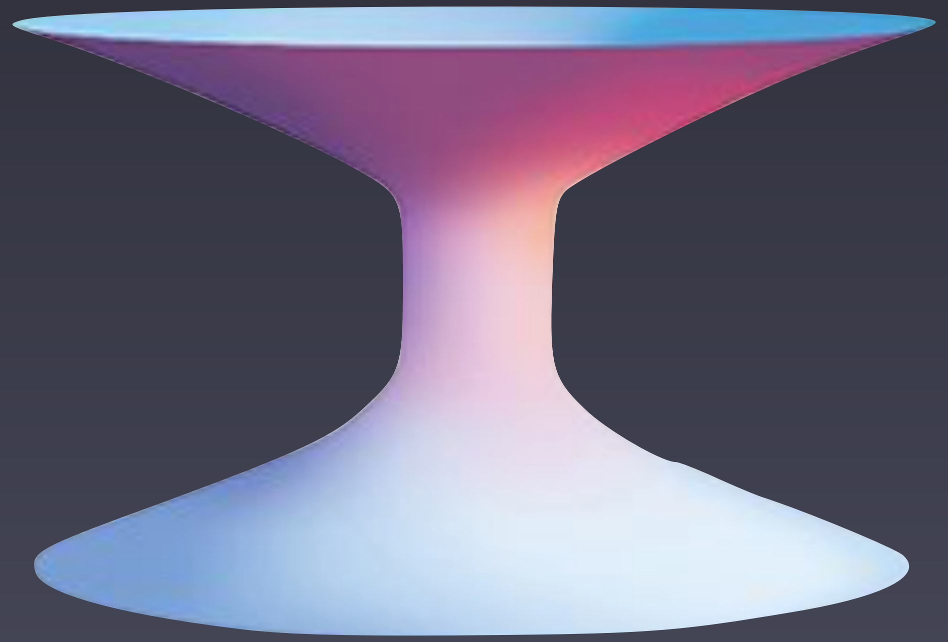
Wormhole in "Interstellar"

- Connect distant parts of universe



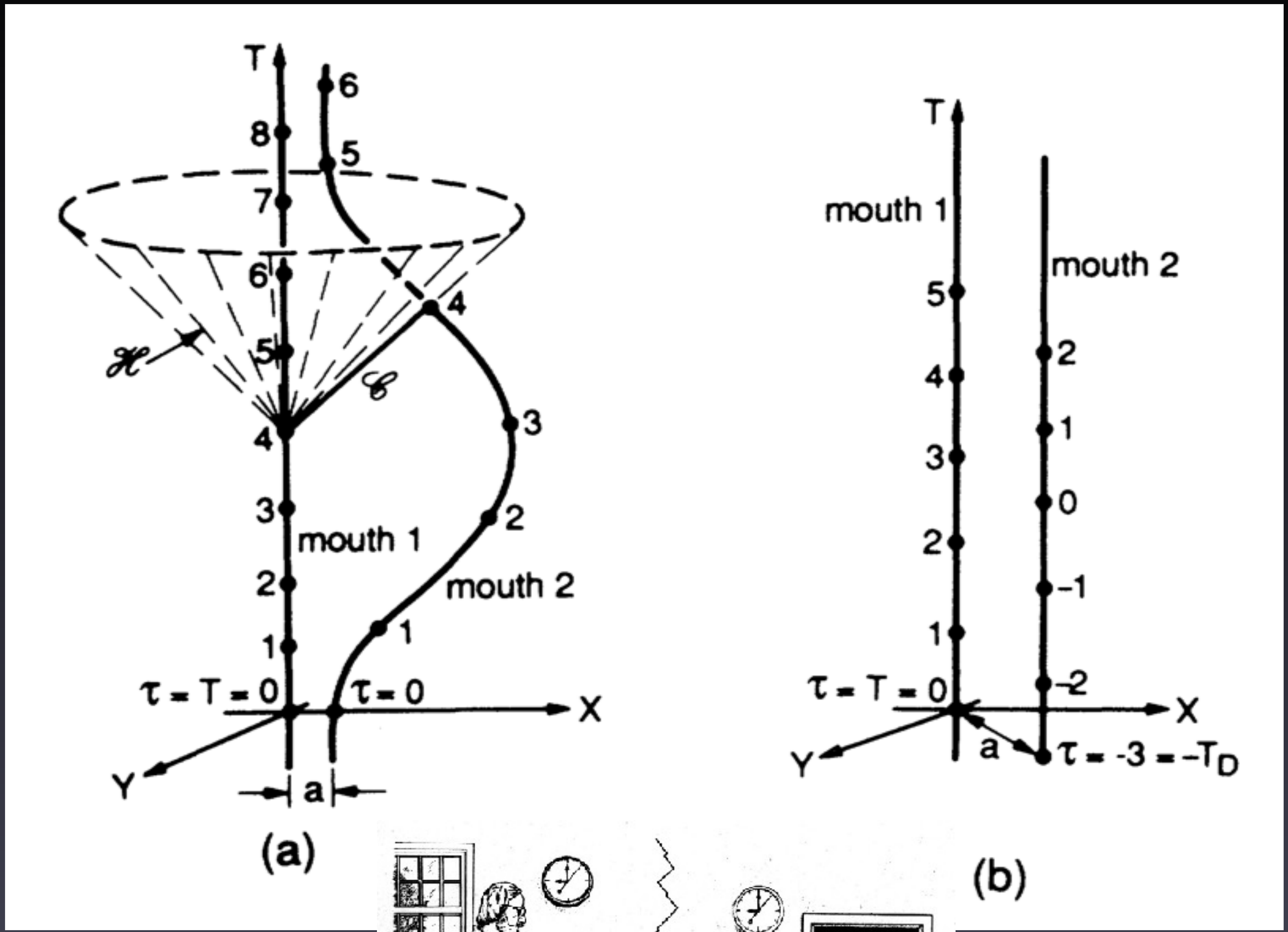


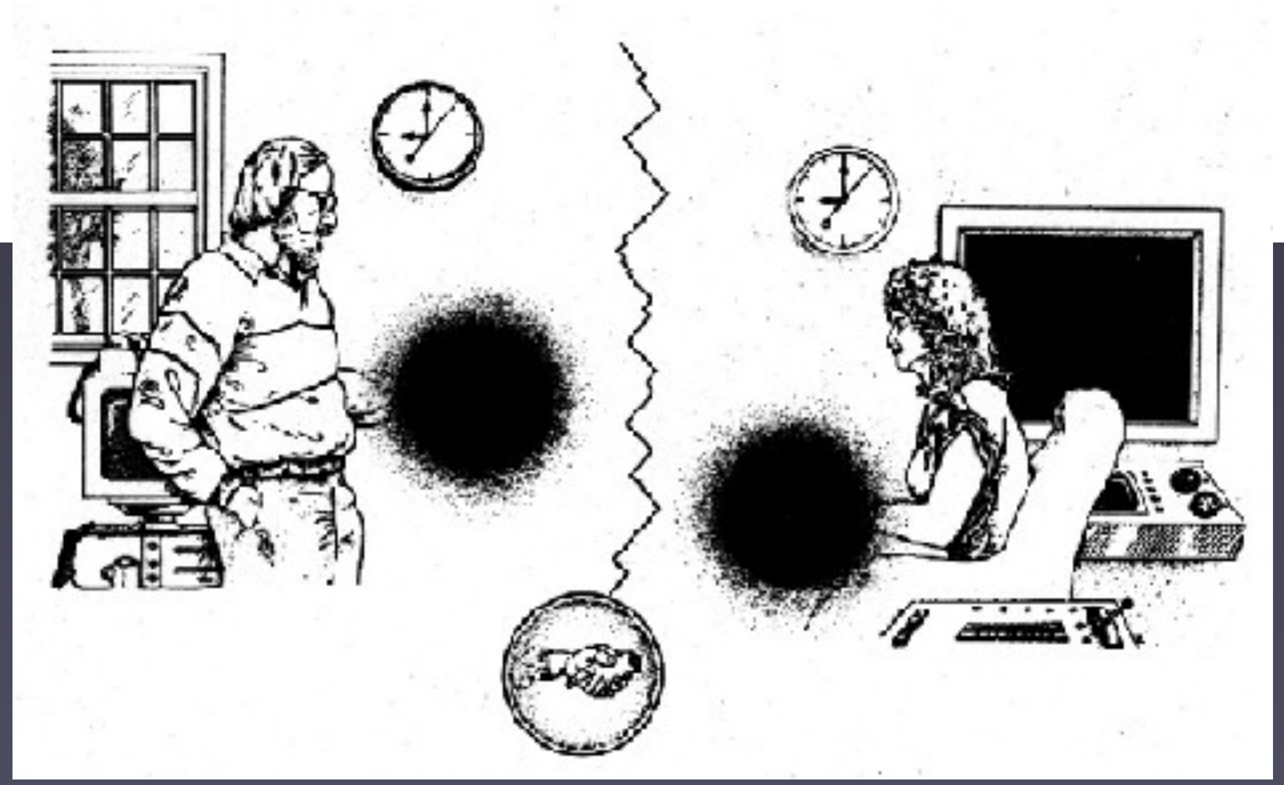
- Wormholes probably can't exist
  - Require "negative mass" to avoid collapsing
- Can be used to make a time machine



Images courtesy Kip Thorne, Paramount, NASA

“Time machine” spacetimes

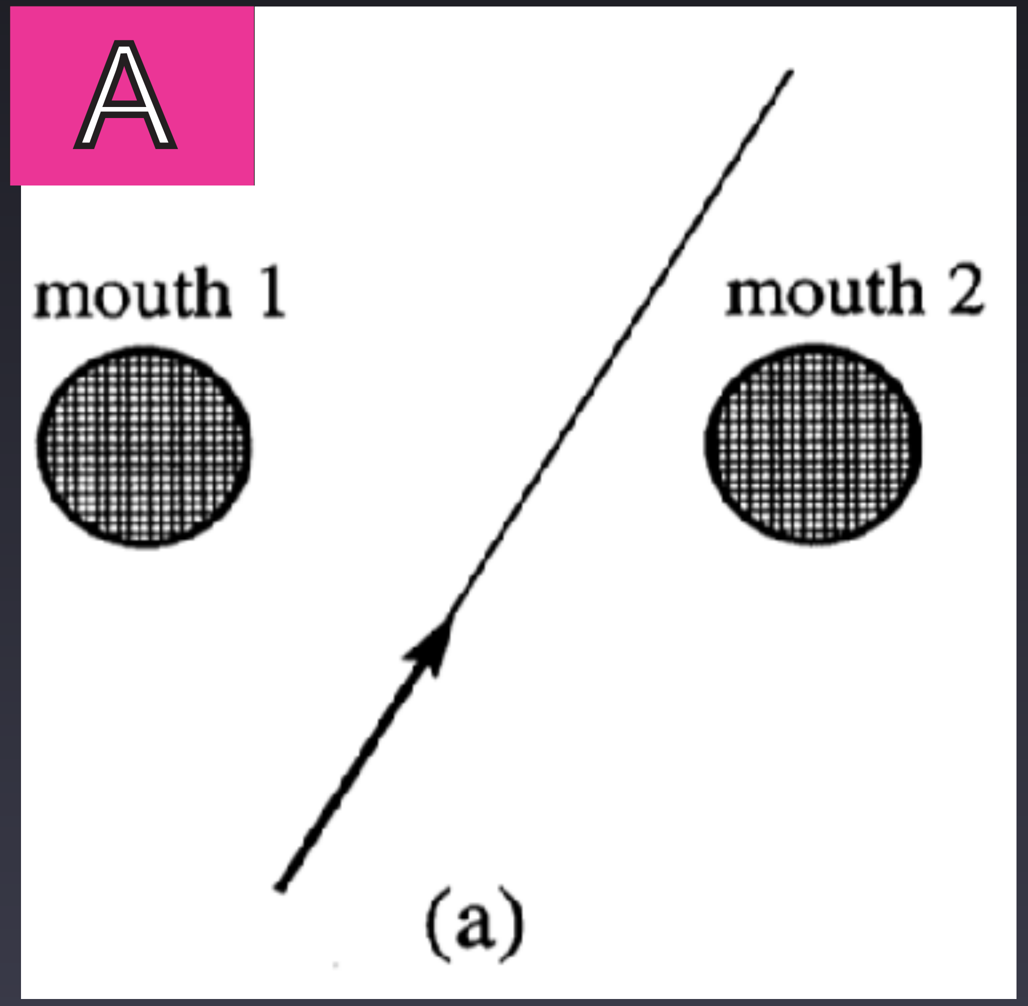


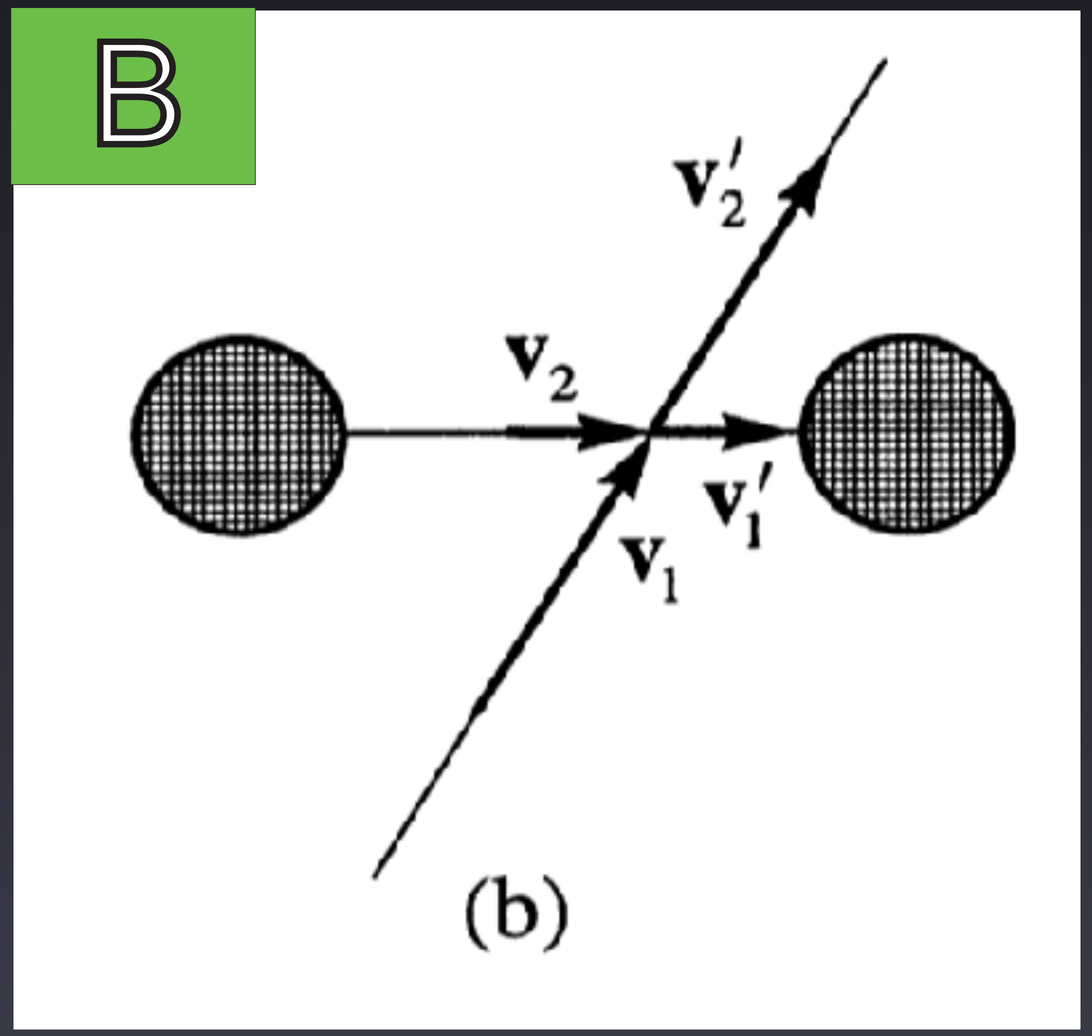


Images courtesy Kip Thorne

What do you think?

- A billiard ball begins with initial velocity velocity $\mathbf{v}_1$, heading between the two mouths of the time machine. Aside from the time machine, Newton's laws of motion apply. What happens?





C

Can't say: both A and B satisfy Newton's laws of motion

```
cd $HOME
cd StudentFolders
cd YOURNAME # replace YOURNAME with the name of your folder
```

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mkdir BlackHoleMerger
cd BlackHoleMerger
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nano Params.input
Omega0 = 0.0;
adot0 = 0.0;
D0 = 35.0;
MassRatio = 1.2; #or 1.0, or something in between
@SpinA = (0.0, 0.0, 0.0); #can make 1 component up to 0.2 instead of
0.1
@SpinB = (0.0, 0.0, 0.0);
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nano Ev/DoMultipleRuns.input
# my MaxLev = 1
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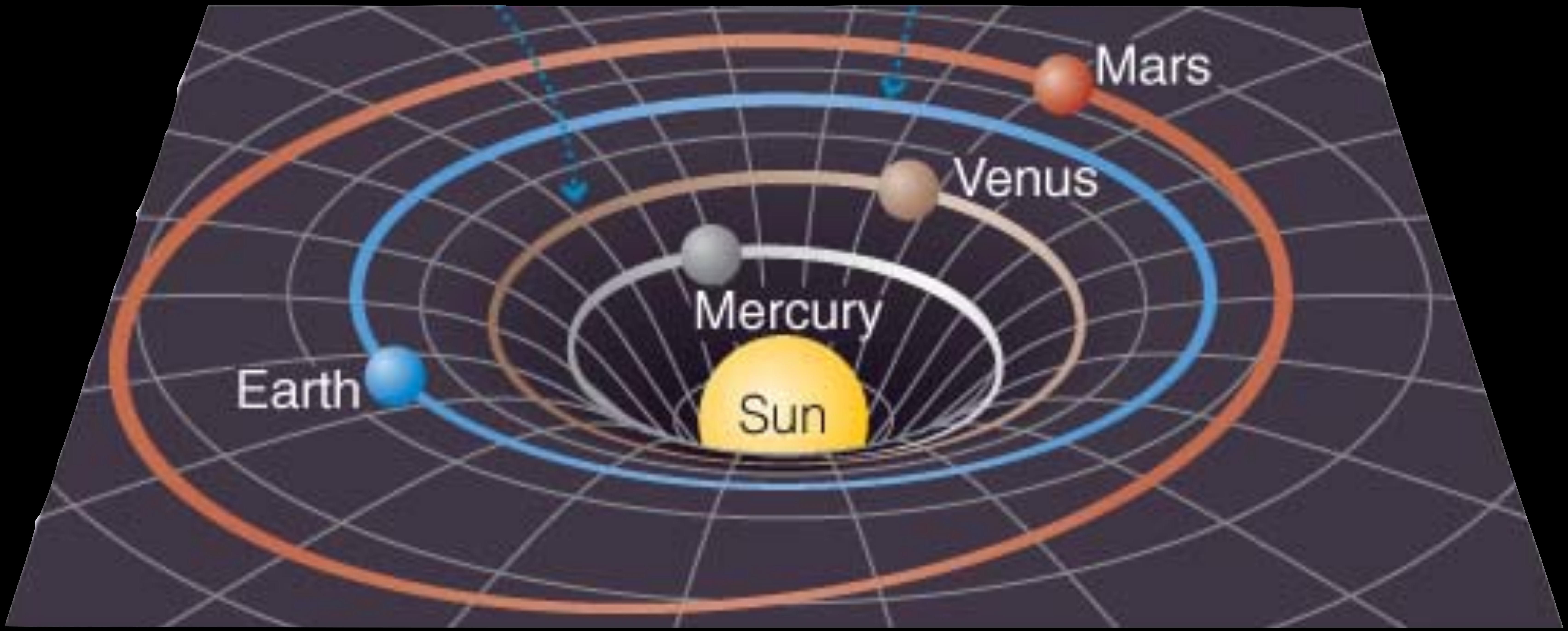
nano Ev/DoMultipleRuns.input
# my MaxLev = 1
./StartJob.sh
```

```
squeue
scontrol show jobid -dd YOUR_JOB_ID
ShowQueue
```

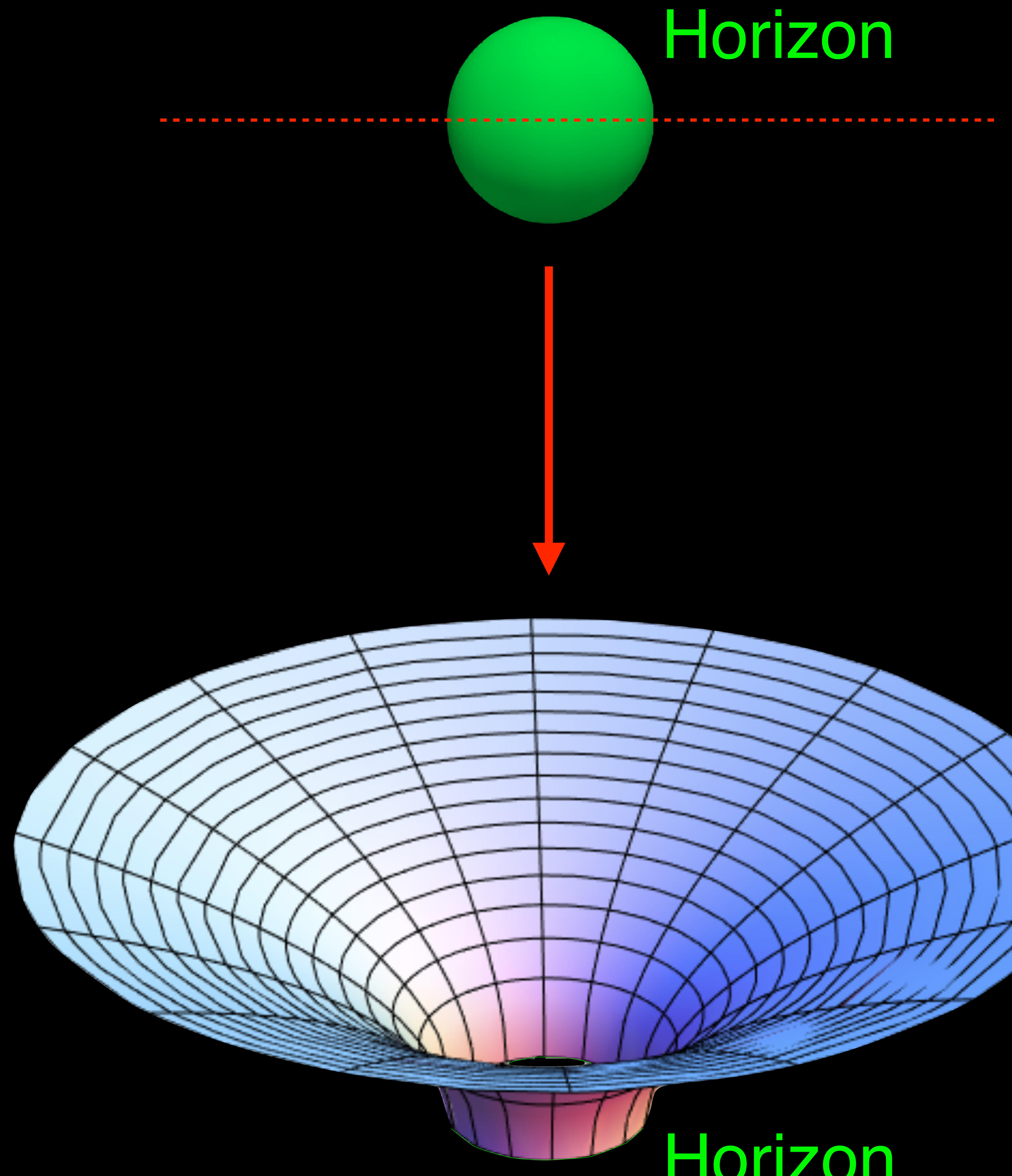
Curved spacetime

"Matter tells spacetime how to curve and space-time tells matter how to move."
- John Wheeler



Extremely curved spacetime: black holes

- Gravity so strong...
  - Nothing (even light) can escape from inside hole's horizon (surface)
  - Singularity inside horizon: *infinitely* strong gravity
- Formed when the most massive stars die



What is spacetime?

- 3 dimensions of space + 1 dimension of time
- Event = a specific place at a specific time

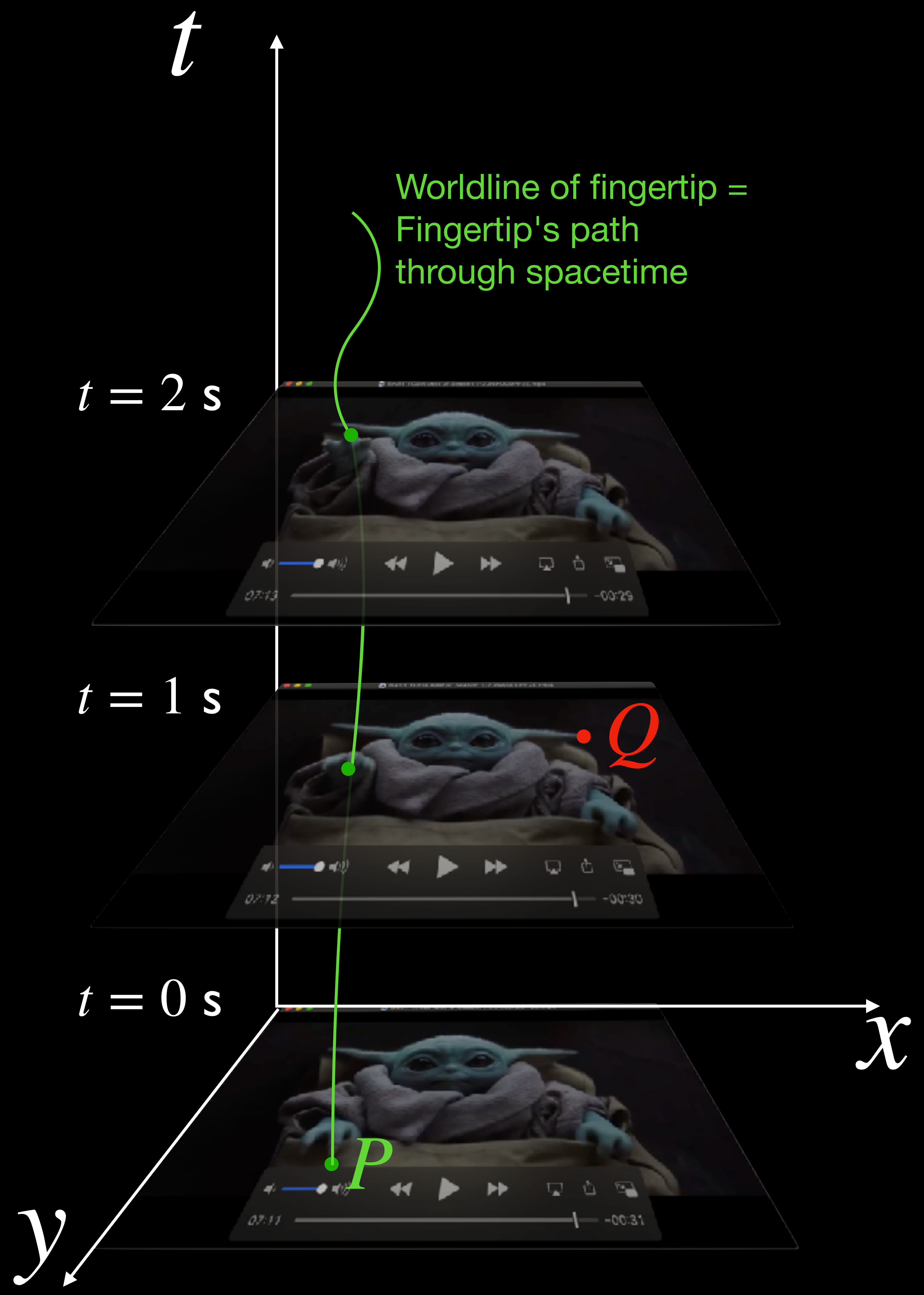
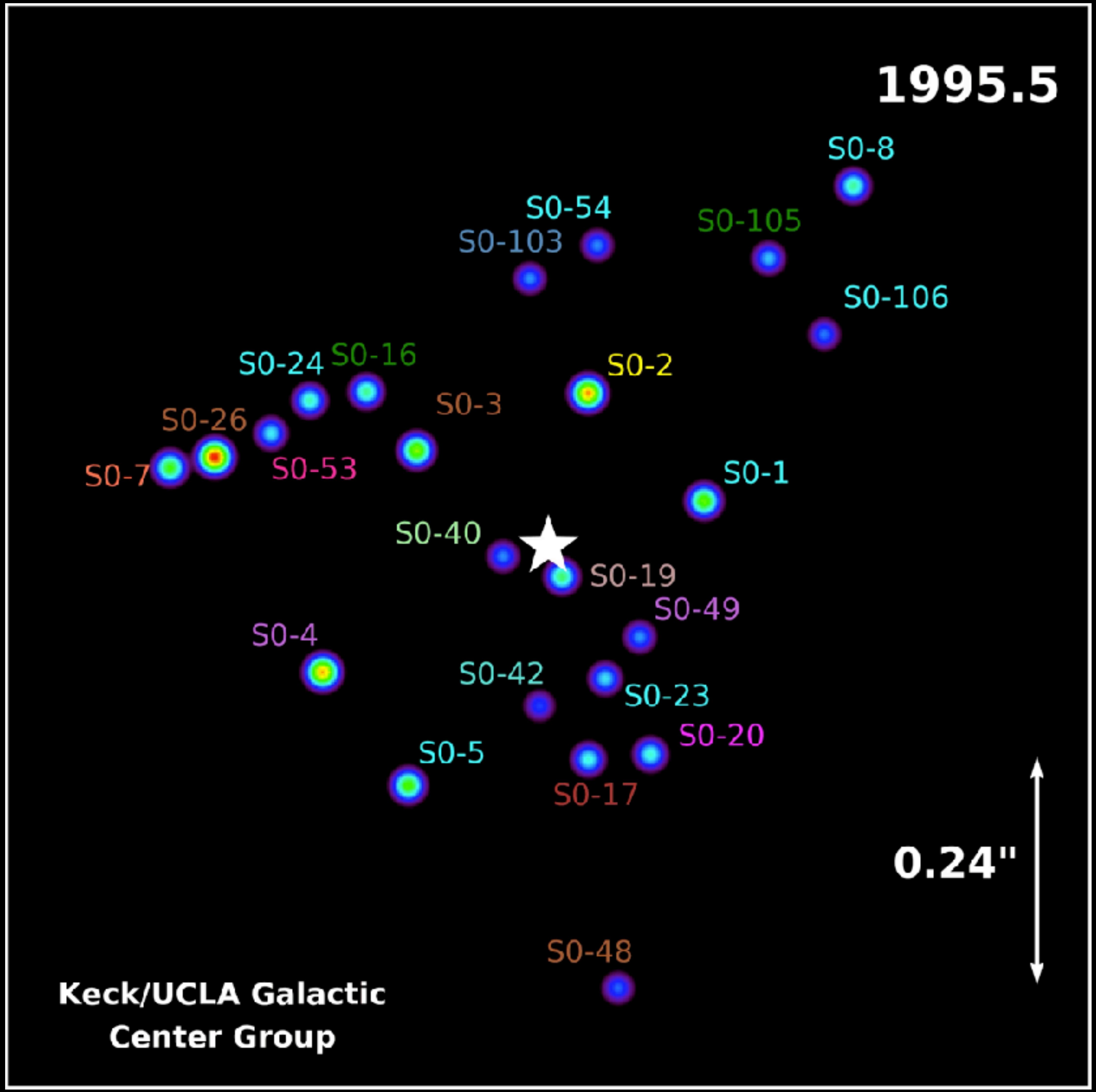


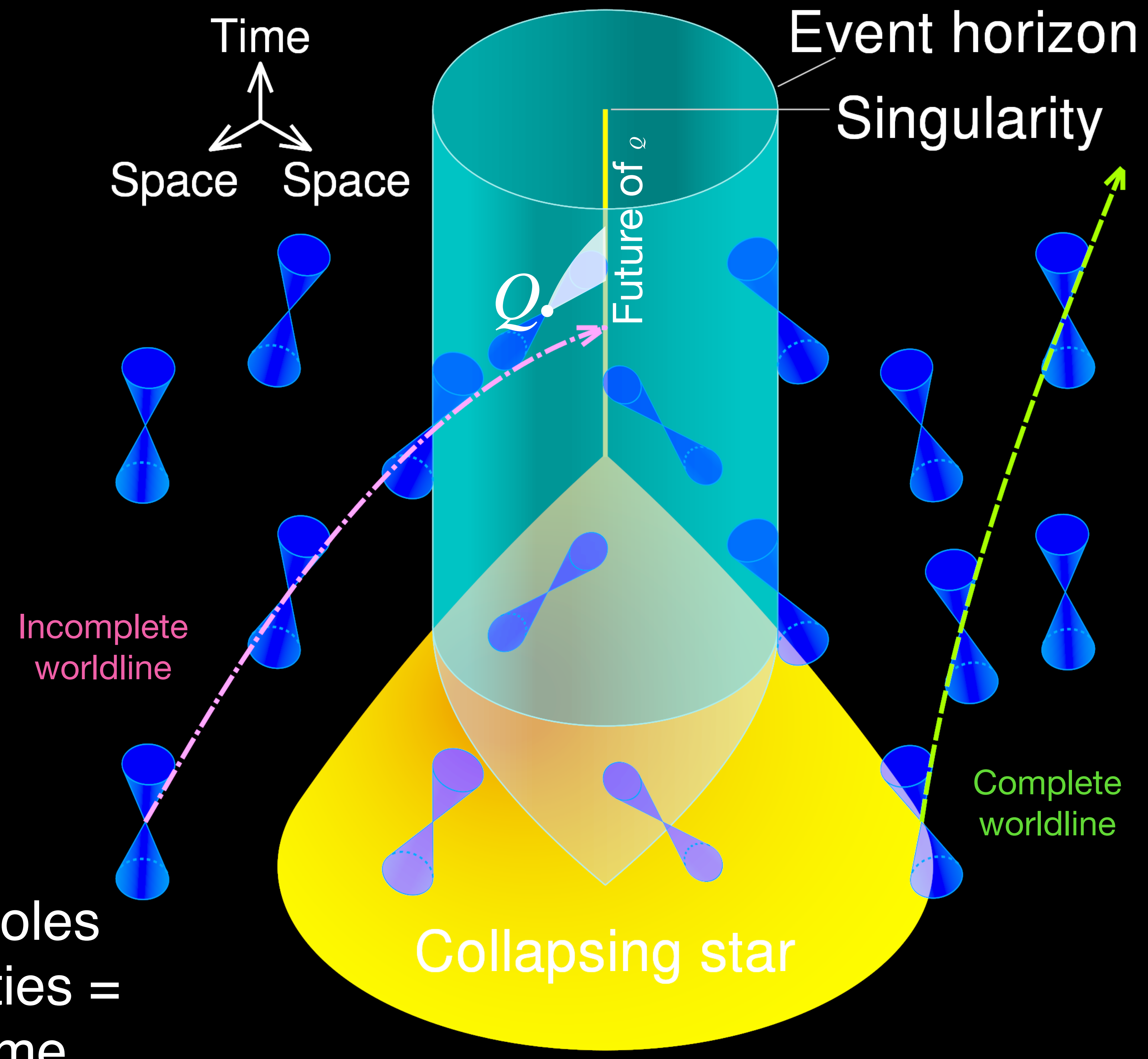
Image & movie courtesy Disney / *The Mandalorian*


Car's future
Events car can reach
moving slower than light
"Time"
Scaled so light travels
on 45° lines
Light's worldline
Car's worldline
"Space"
"Nutwood Ave."
CALIFORNIA
57
50TH ANNIVERSARY
Habit
Photo courtesy Mariah Sanchez, Daily Titan; images courtesy google, wikipedia

2020 Nobel Prize in Physics



Genzel & Ghez (local at UCLA): there's a black hole at the center of our galaxy



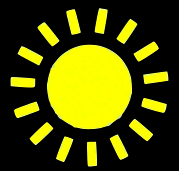
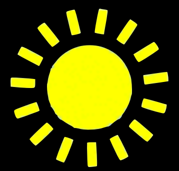
Penrose: black holes contain singularities = edges of spacetime

Reinhard Genzel

Andrea Ghez

Roger Penrose

How big are black holes?

- Mass: huge!
  - Two kinds
  - 3 to ~100 
  - Millions+ 
- Radius: small!

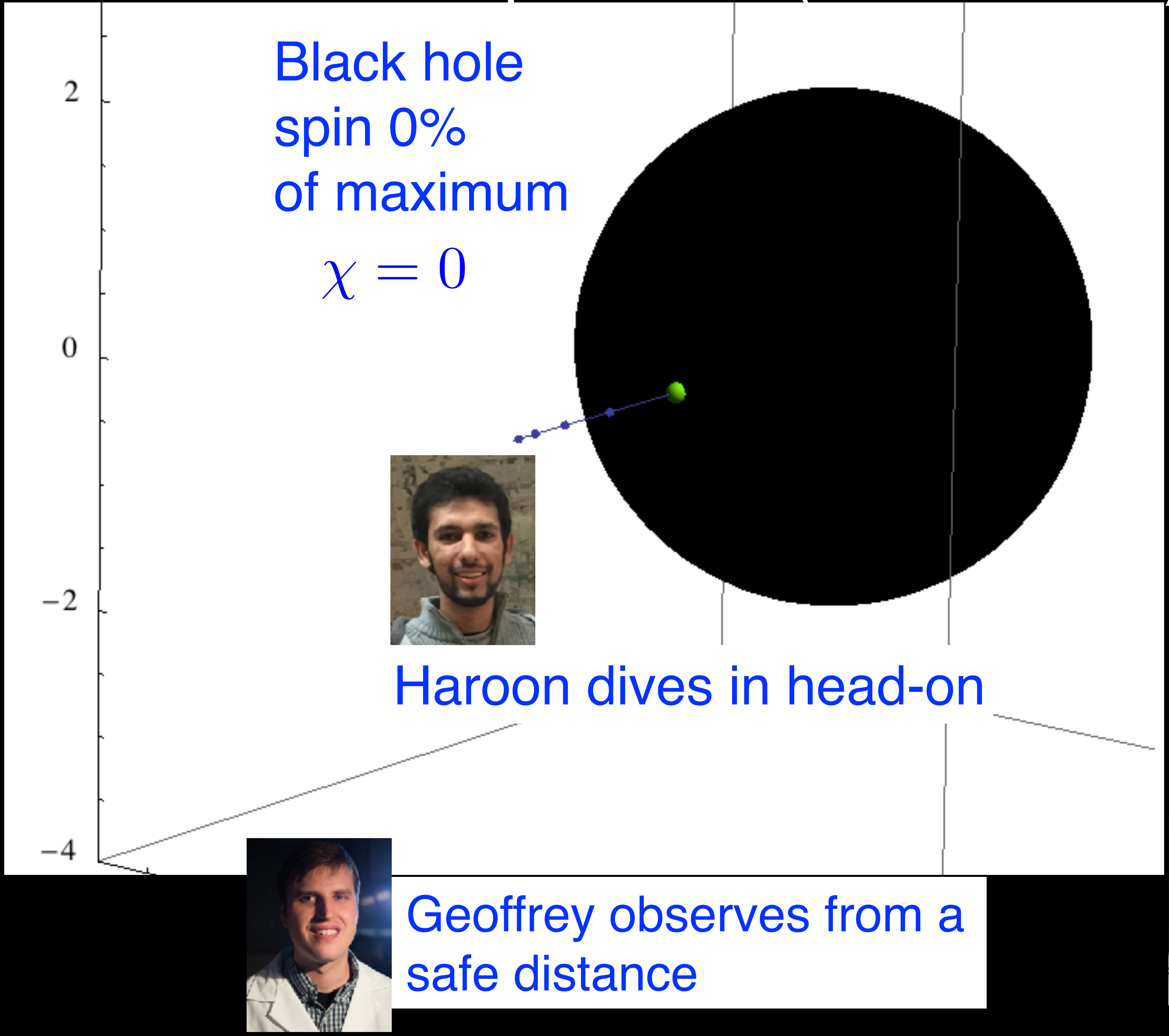
Size of earth-mass black hole

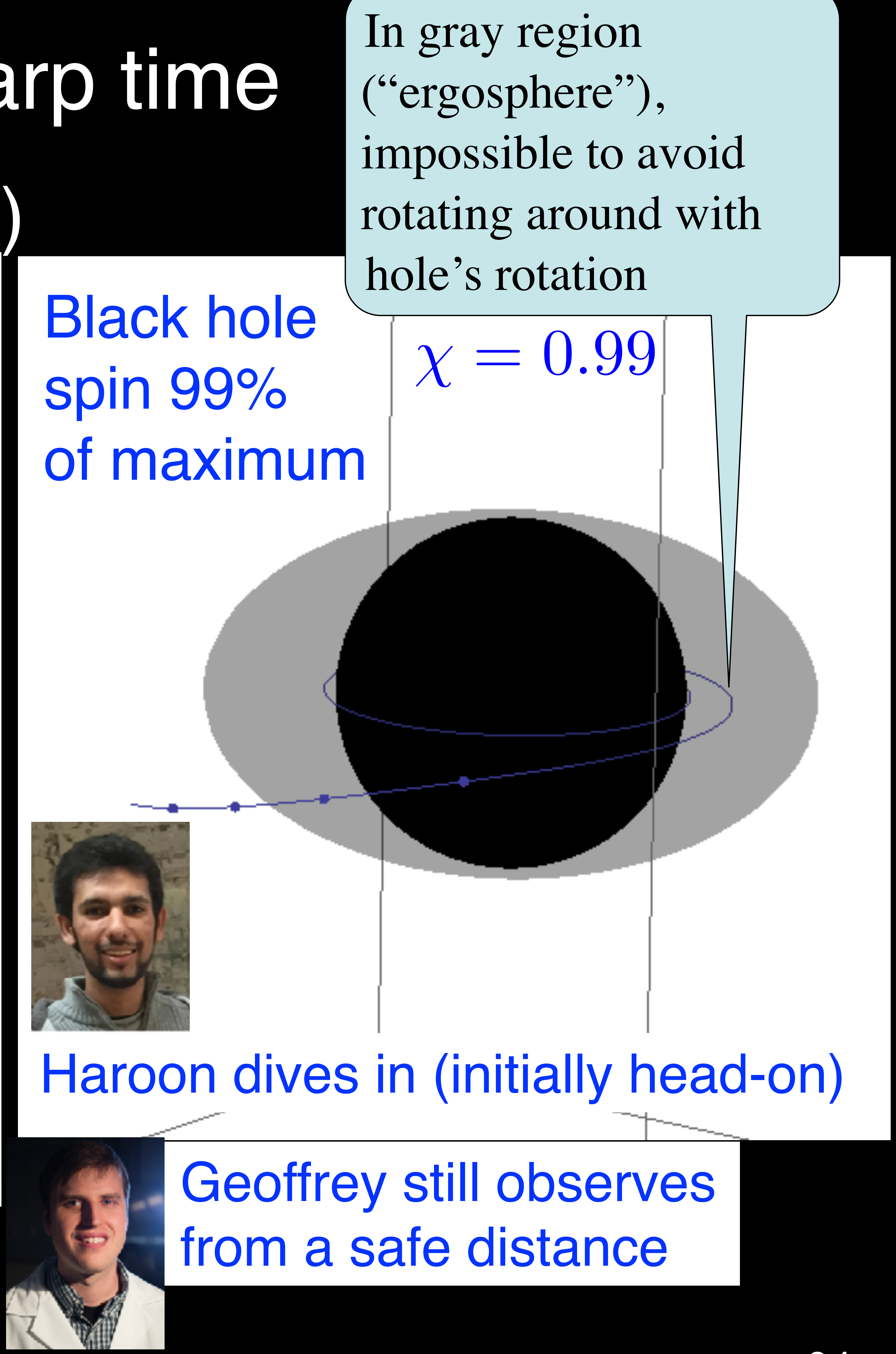
Size of 10- black hole

Map courtesy Apple maps

Black holes rotate and warp time

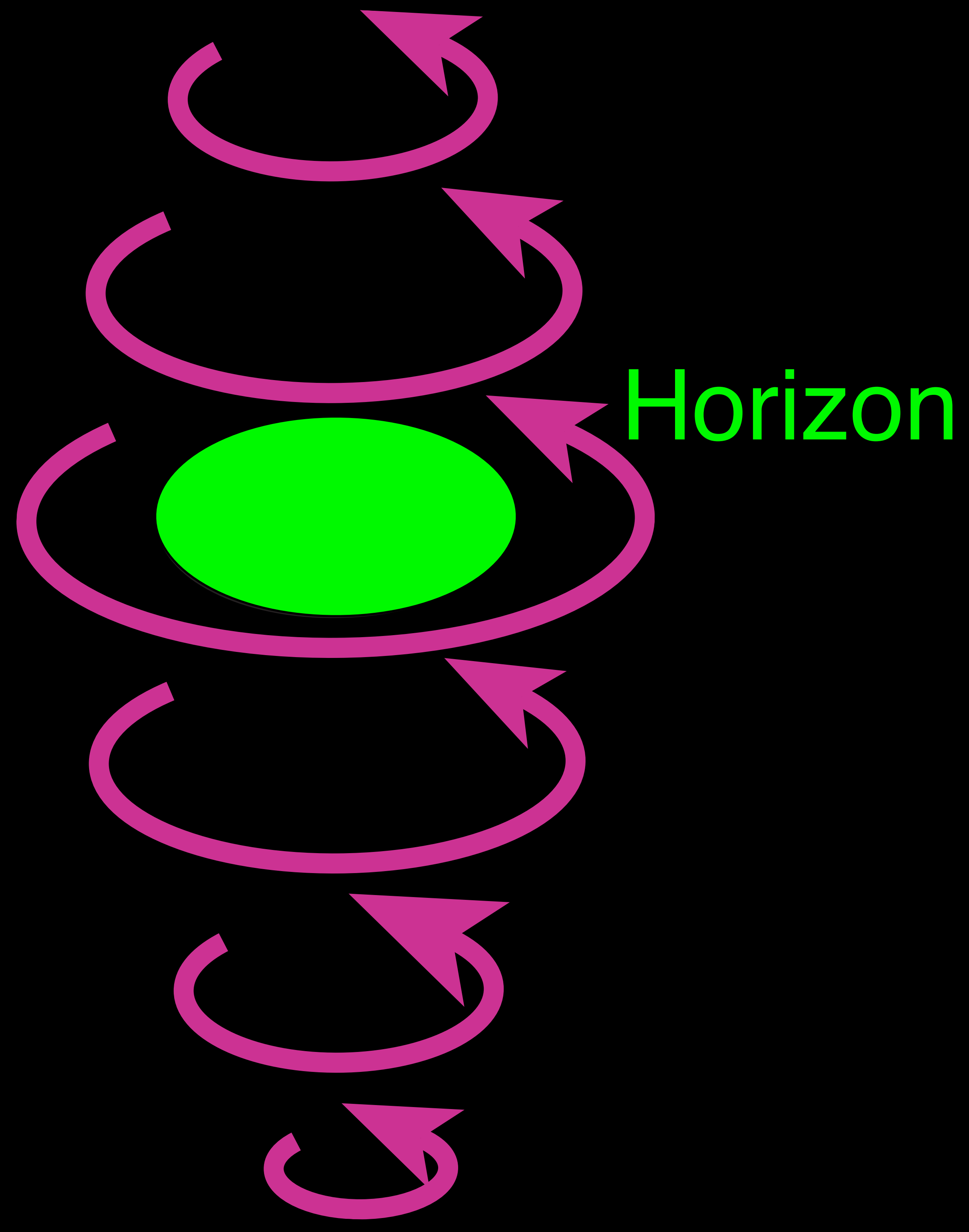
- An experiment (not to scale)

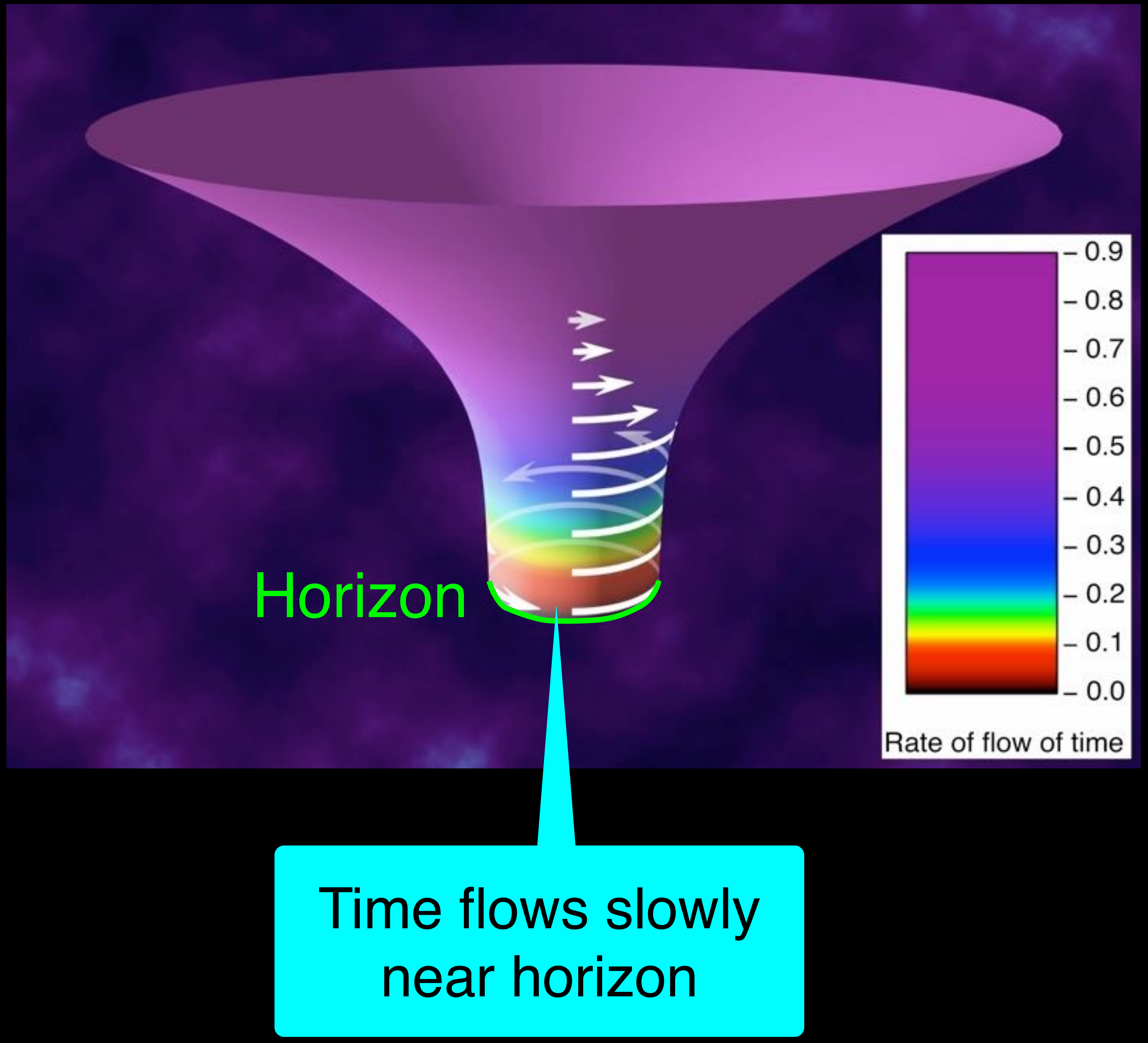




Black holes rotate and warp time

- Whirl space like a tornado





Images courtesy Kip Thorne

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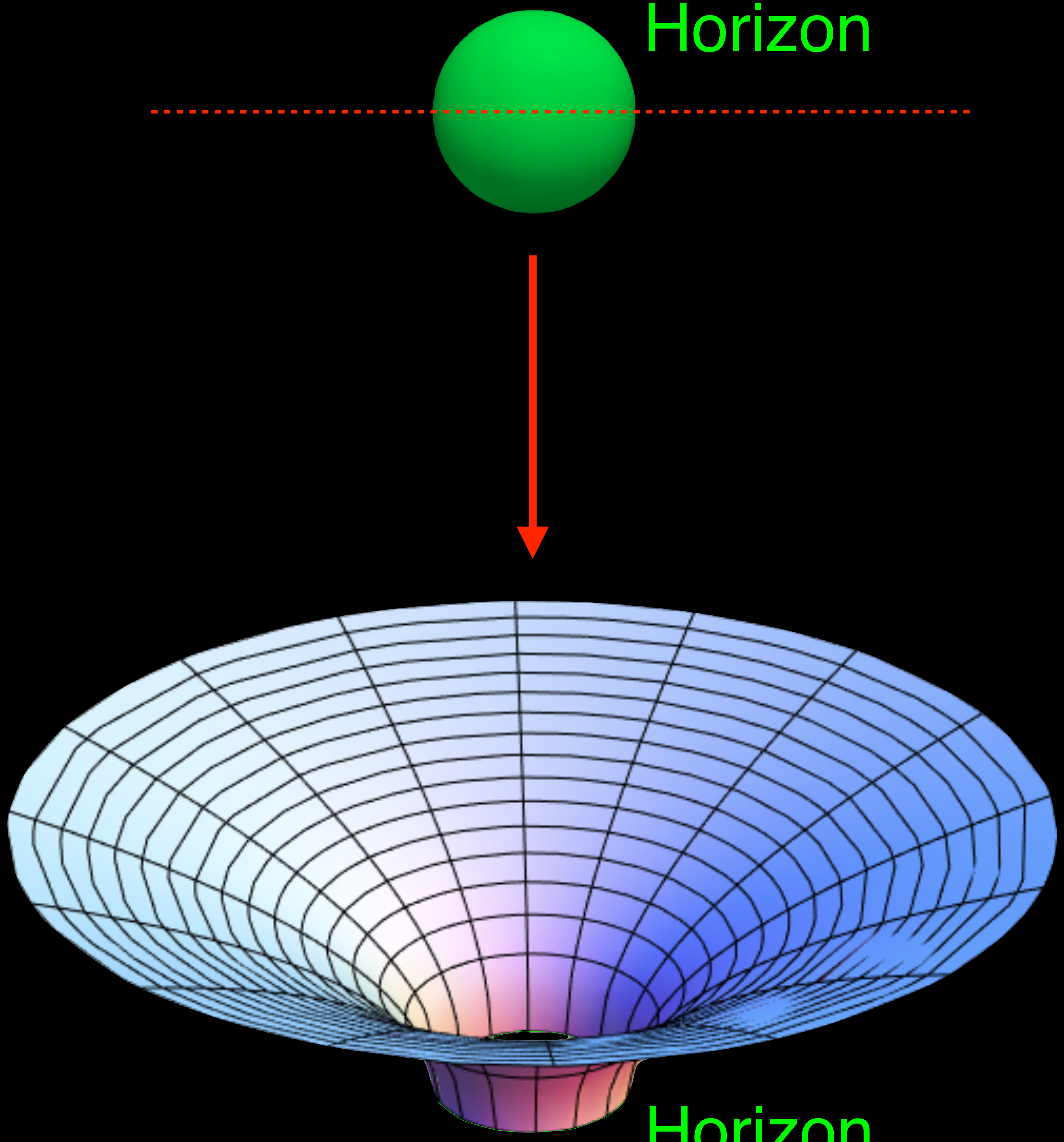

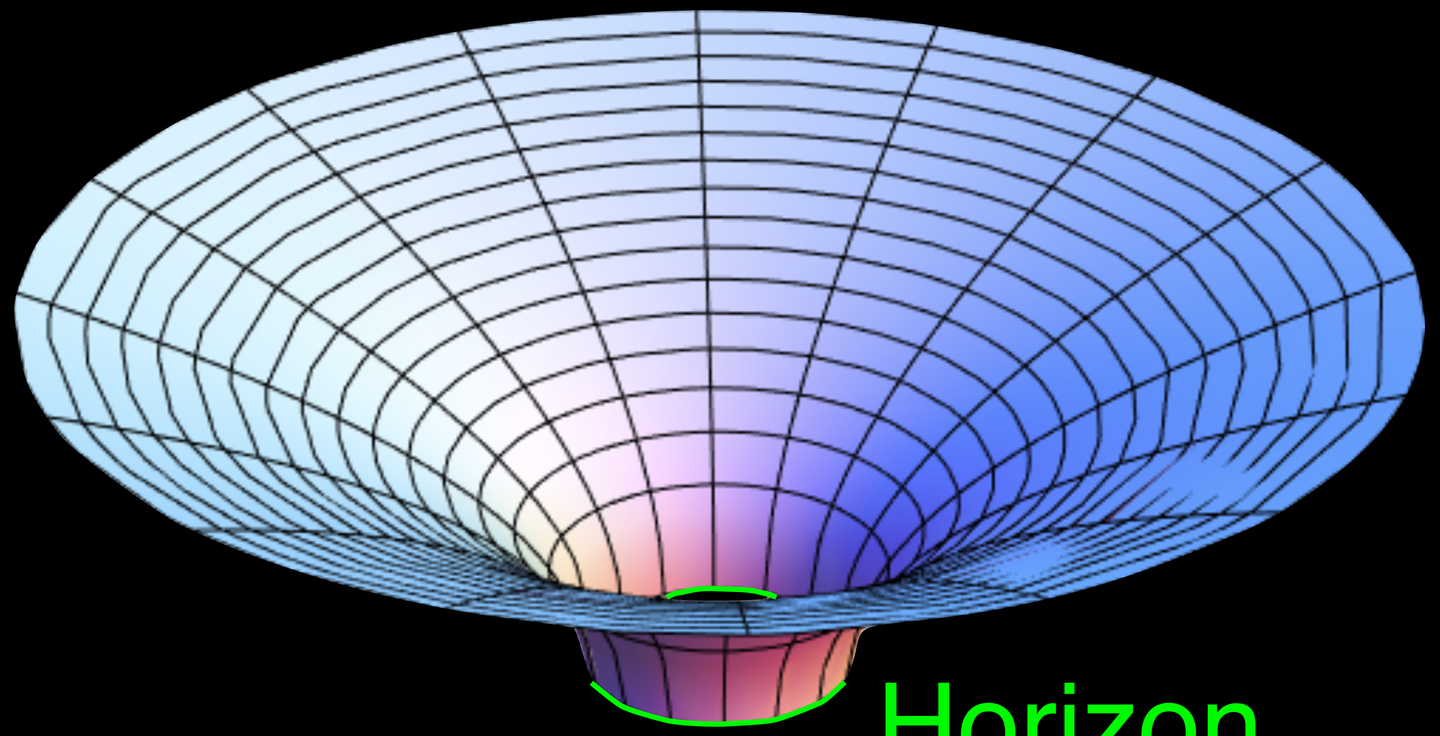
Horizon

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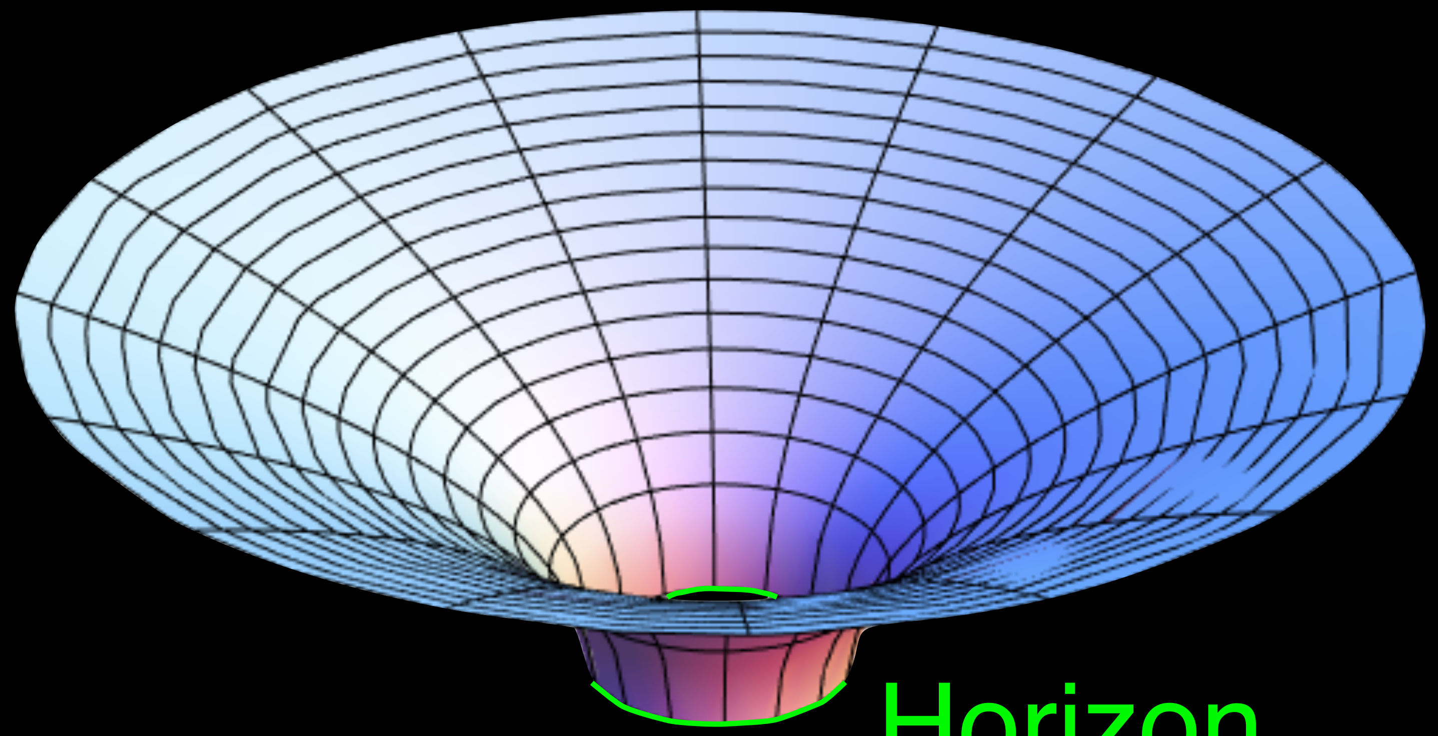

Horizon

= ?

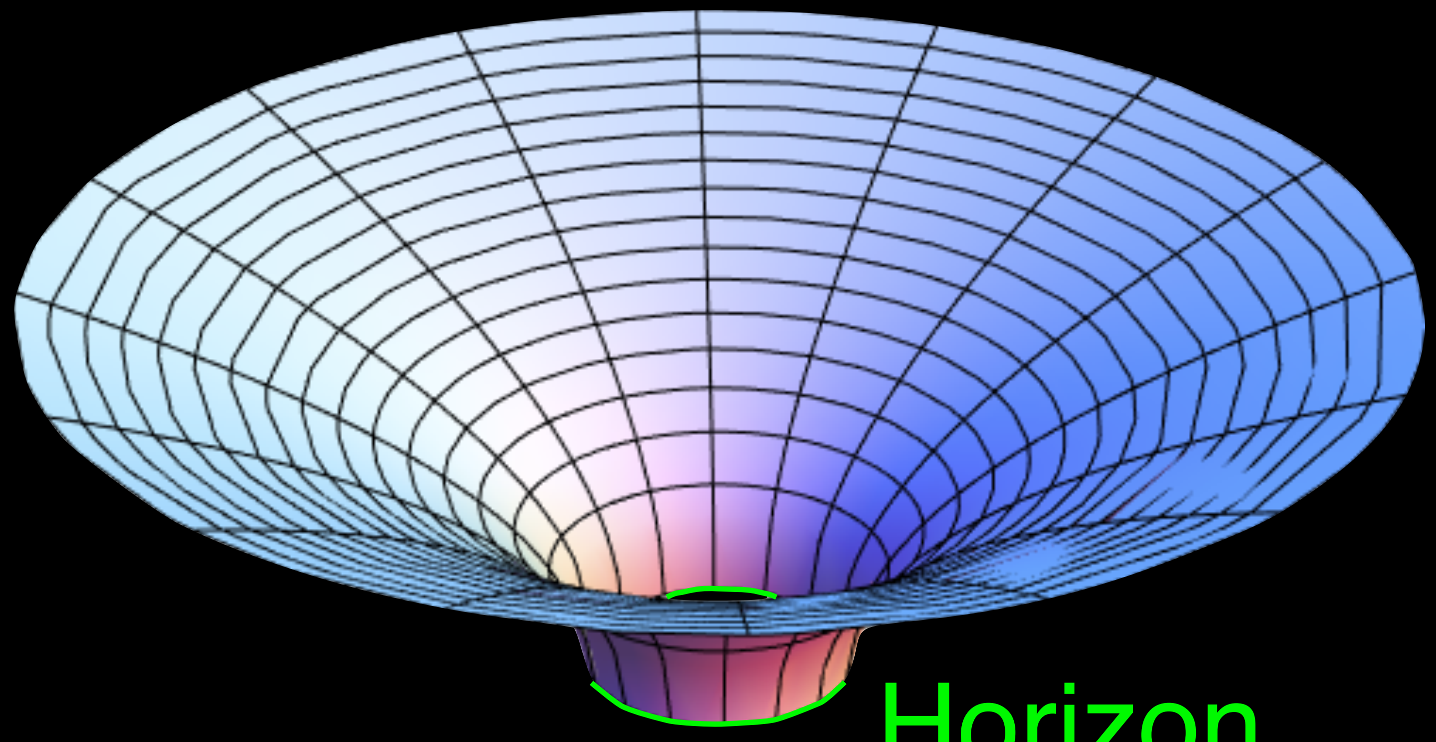
Linear and nonlinear physics

- Linear
  - Whole is sum of parts
  - Example: sound in this room
  - Total sound = sum of individual sounds
- Nonlinear
  - Whole is more than sum of parts
  - Example: water + wind
  - Example: two black holes
  - Need supercomputers to predict

Single black hole

Colliding black holes

Images courtesy Kip Thorne



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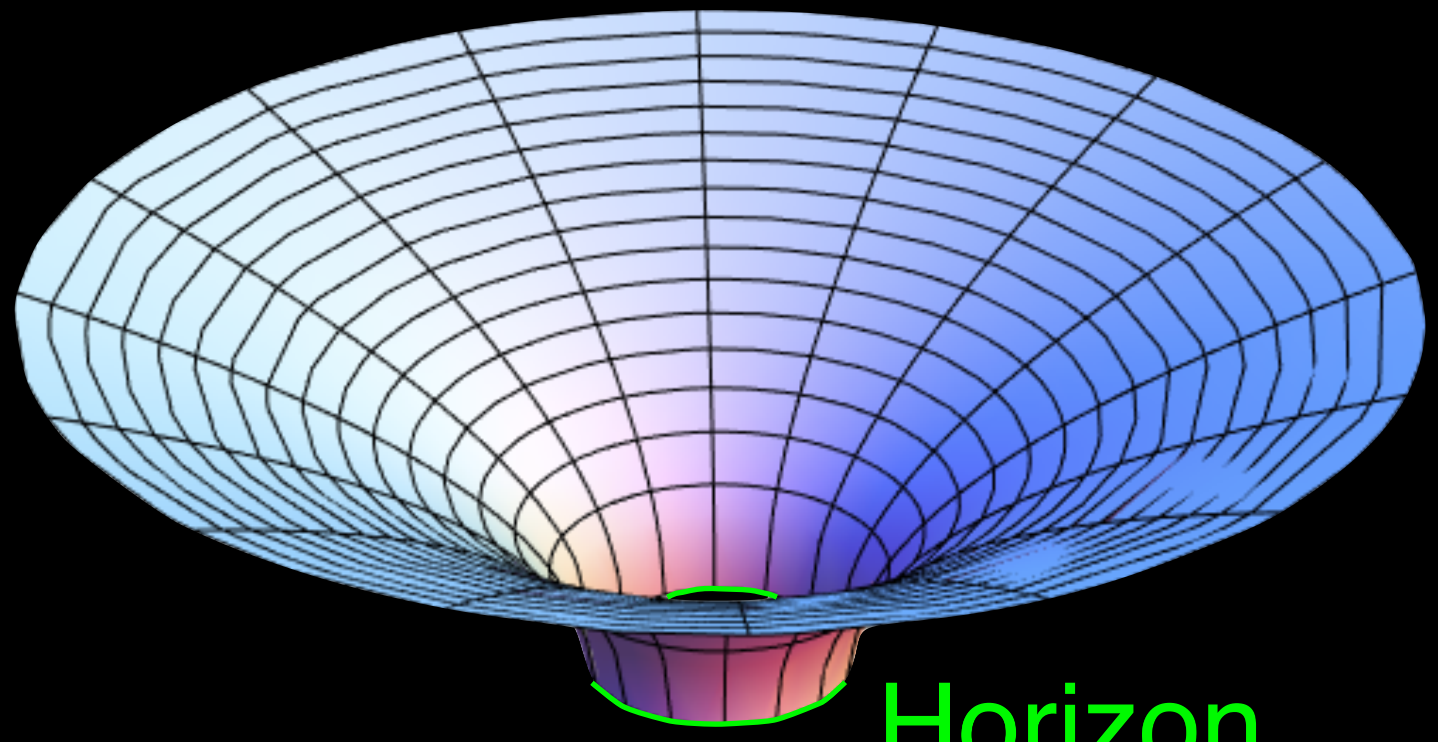

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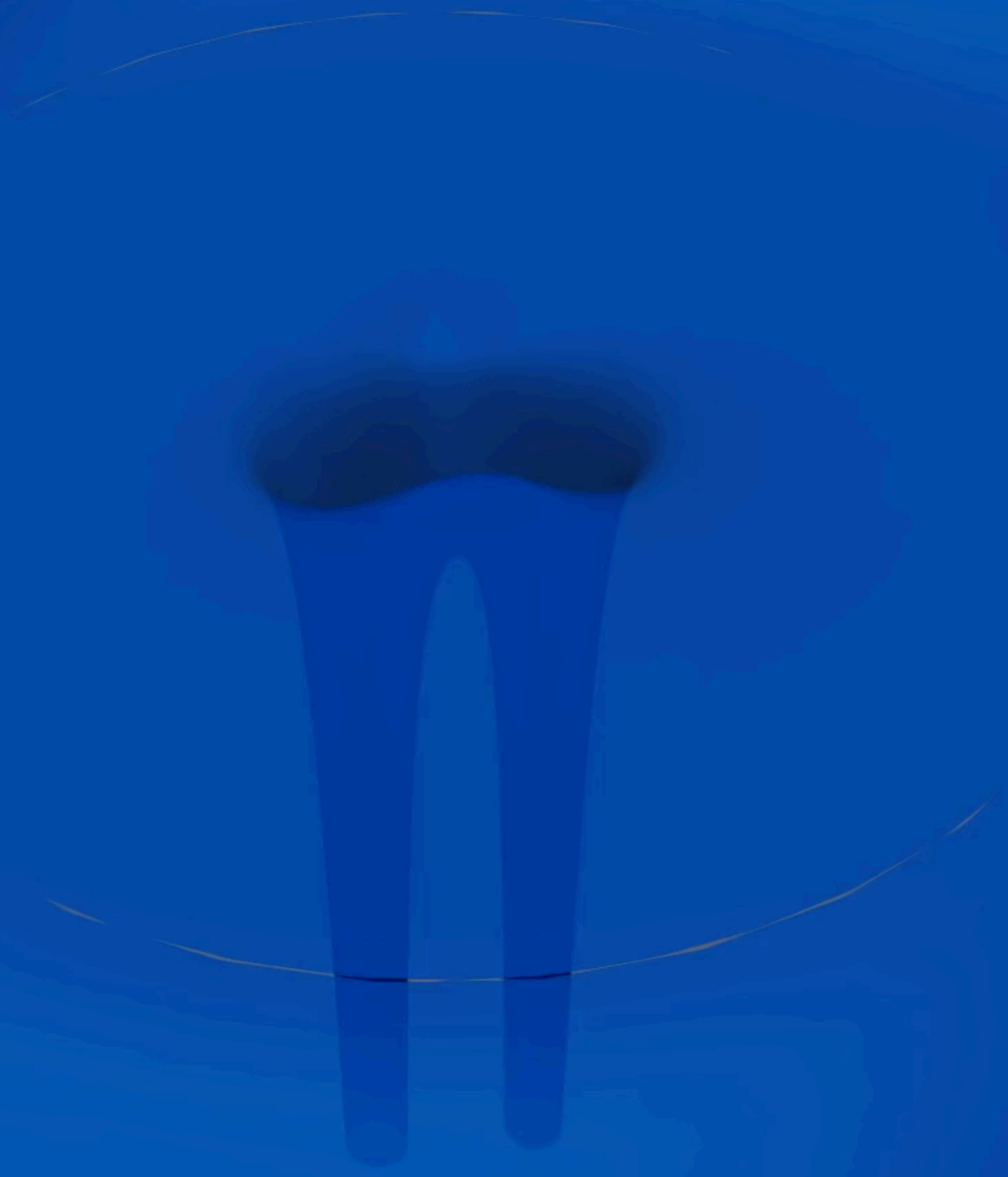
Merging black holes & gravitational waves



By CSUF Undergrad
Nick Demos
(now MIT PhD student)

SXS Collaboration: "Calculation of warped spacetime consistent with GW170104 (zoomed)"

3+1 decomposition

Split spacetime into set of spaces

Spacetime metric = measure of distance between events in spacetime

Goal: evolve (constraint-satisfying) spacetime metric g_{ab}

Spacetime metric $g_{ab}(t = 0)$

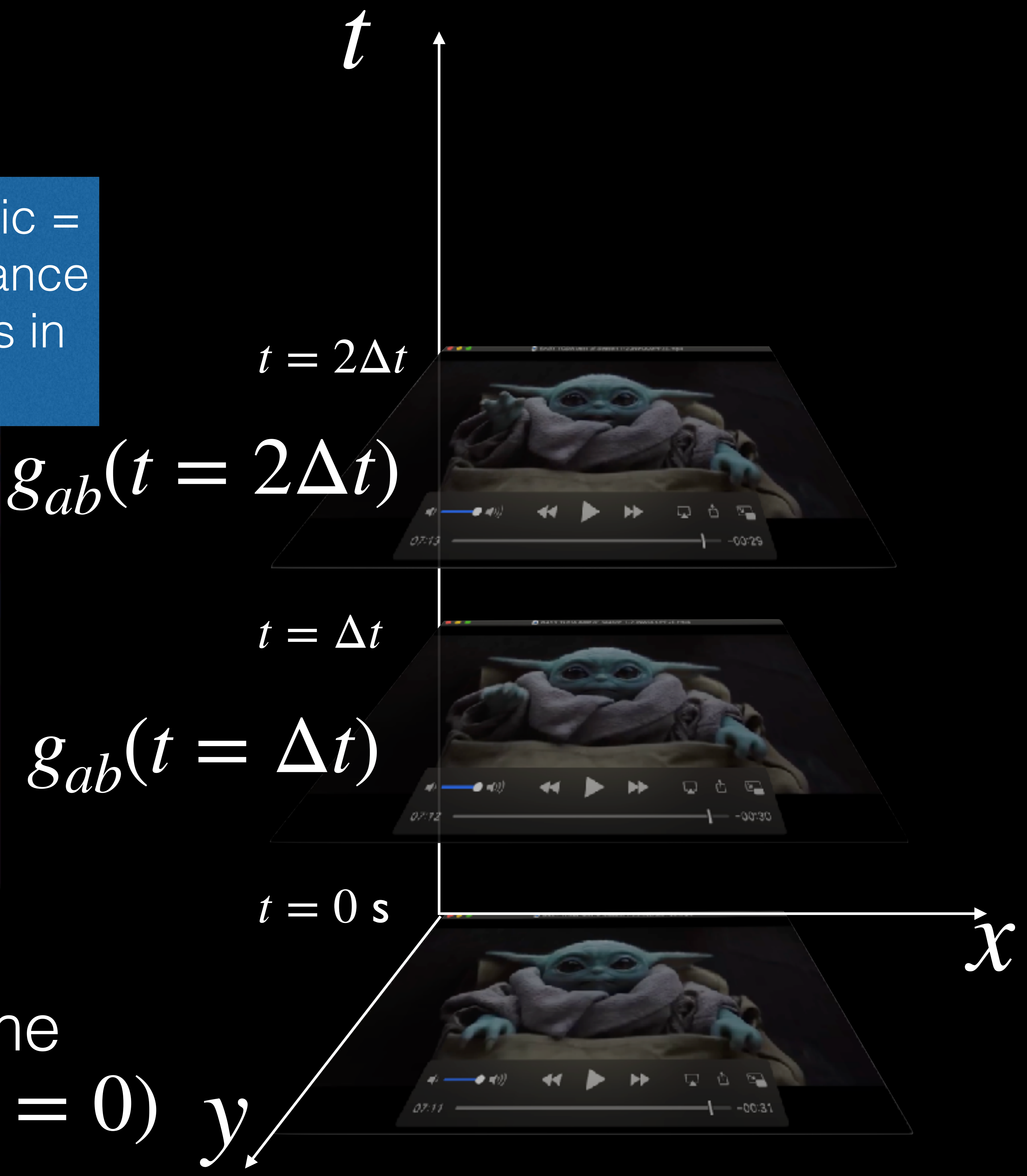
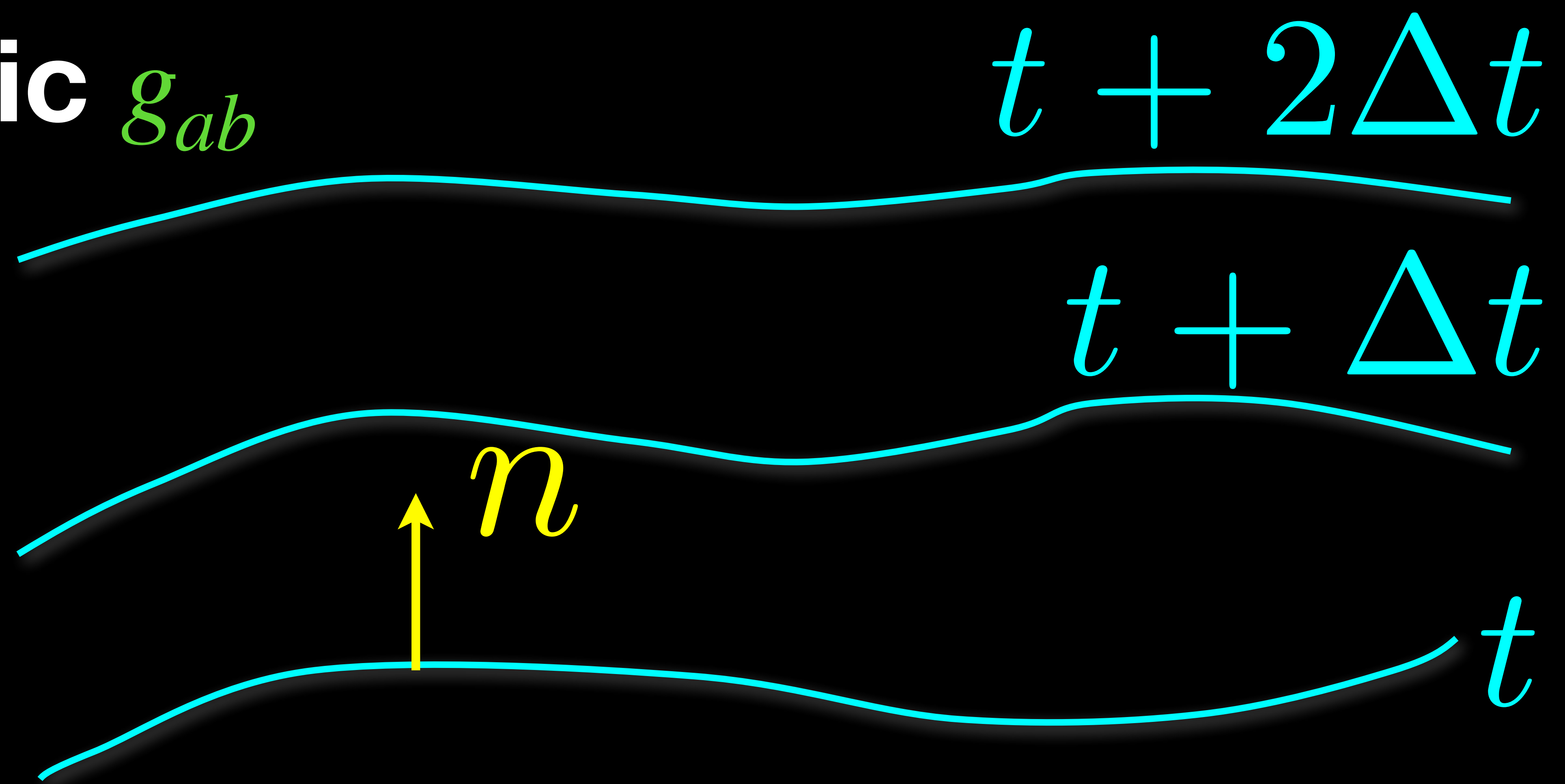


Image & movie courtesy Disney / *The Mandalorian*

Solving Einstein's equations in vacuum

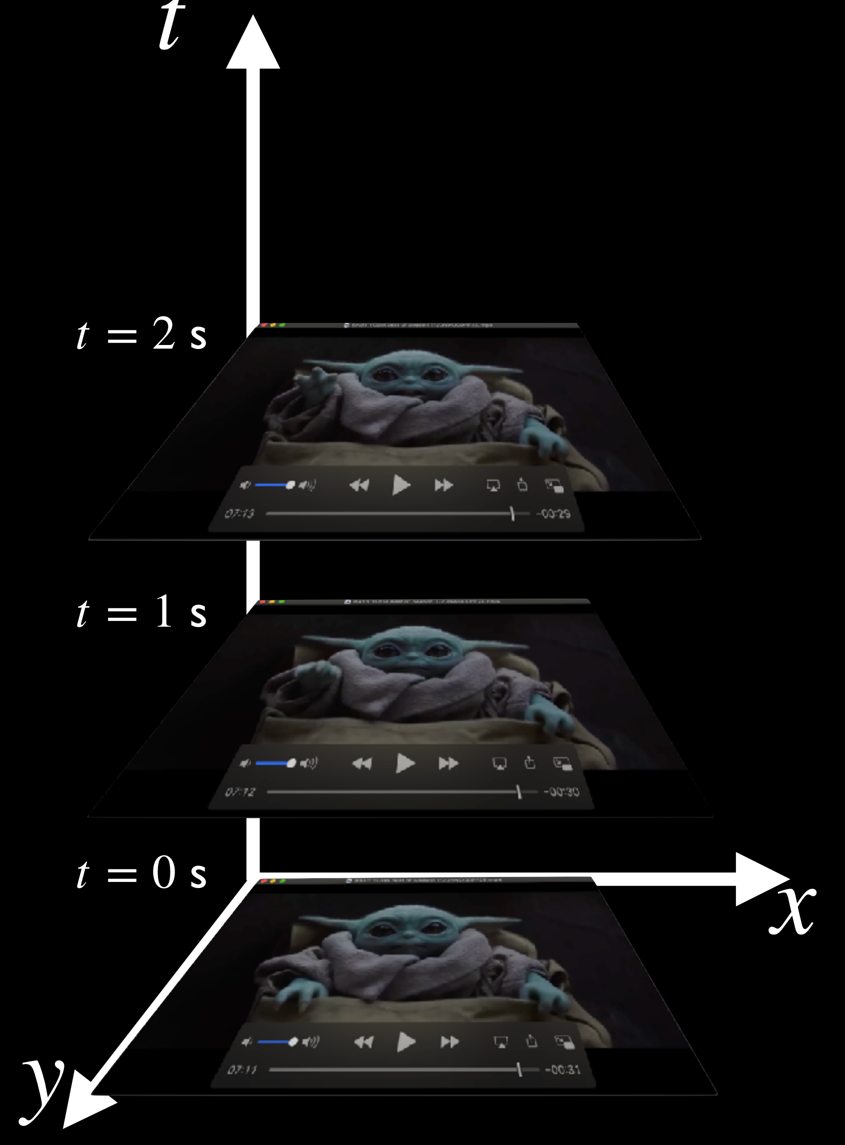
Goal: solve $G_{\mu\nu} = 0$ **for spacetime metric** g_{ab}



- Split spacetime into space + time
  - Constraint equations $G_{nj} = 0 \quad G_{nn} = 0$
    - Solve to create initial data like $\nabla \cdot E = 0, \nabla \cdot B = 0$
  - Evolution equations $G_{ij} = 0$
    - Constraints must stay satisfied

like $$\frac{\partial B}{\partial t} = -\nabla \times E,$$
$$\frac{\partial E}{\partial t} = \nabla \times B$$

- Step 1: Step forward in time
- Step 2: Repeat step 1 (a lot)



The actual equations we solve

$$
\begin{aligned}
&\partial_t g_{ab} - (1+\gamma_1)\beta^k\partial_k g_{ab} = -\alpha\Pi_{ab} - \gamma_1\beta^i\Phi_{iab},\\
&\partial_t\Pi_{ab} - \beta^k\partial_k\Pi_{ab} + \alpha\gamma^{ki}\partial_k\Phi_{iab} - \gamma_1\gamma_2\beta^k\partial_k g_{ab}\\
&\quad = 2\alpha g^{cd}\left(\gamma^{ij}\Phi_{ica}\Phi_{jdb} - \Pi_{ca}\Pi_{db} - g^{ef}\Gamma_{ace}\Gamma_{bdf}\right)\\
&\qquad - 2\alpha\nabla_{(a}H_{b)} - \frac{1}{2}\alpha n^c n^d\Pi_{cd}\Pi_{ab} - \alpha n^c\Pi_{ci}\gamma^{ij}\Phi_{jab}\\
&\qquad + \alpha\gamma_0\left(2\delta^c{}_{(a}n_{b)} - (1+\gamma_3)g_{ab}n^c\right)\mathcal{C}_c\\
&\qquad + 2\gamma_4\alpha\Pi_{ab}n^c\mathcal{C}_c\\
&\qquad - \gamma_5\alpha n^c\mathcal{C}_c\left(\frac{\mathcal{C}_a\mathcal{C}_b - \frac{1}{2}g_{ab}\mathcal{C}_d\mathcal{C}^d}{\epsilon_5 + 2n^d\mathcal{C}_d n^e\mathcal{C}_e + \mathcal{C}_d\mathcal{C}^d}\right)\\
&\qquad - \gamma_1\gamma_2\beta^i\Phi_{iab}\\
&\qquad - 16\pi\alpha\left(T_{ab} - \frac{1}{2}g_{ab}T^c{}_c\right),\\
&\partial_t\Phi_{iab} - \beta^k\partial_k\Phi_{iab} + \alpha\partial_i\Pi_{ab} - \alpha\gamma_2\partial_i g_{ab}\\
&\quad = \frac{1}{2}\alpha n^c n^d\Phi_{icd}\Pi_{ab} + \alpha\gamma^{jk}n^c\Phi_{ijc}\Phi_{kab}\\
&\qquad - \alpha\gamma_2\Phi_{iab},
\end{aligned}
$$

Evolution equations $u_\alpha = \{g_{ab}, \Pi_{ab}, \Phi_{iab}\}$

$$\partial_t u_\alpha + \partial_i F^i_\alpha + B^i_{\alpha\beta}\partial_i u_\beta - S_\alpha = 0.$$

$$C_a = H_a + g^{ij}\Phi_{ija} + t^b\Pi_{ba} - \frac{1}{2}g^i_a\psi^{bc}\Phi_{ibc} - \frac{1}{2}t_a\psi^{bc}\Pi_{bc}$$

$$
\begin{aligned}
C_{ia} \equiv\ & g^{jk}\partial_j\Phi_{ika} - \frac{1}{2}g^j_a\psi^{cd}\partial_j\Phi_{icd} + t^b\partial_i\Pi_{ba} - \frac{1}{2}t_a\psi^{cd}\partial_i\Pi_{cd}\\
& + \partial_i H_a + \frac{1}{2}g^j_a\Phi_{jcd}\Phi_{ief}\psi^{ce}\psi^{df} + \frac{1}{2}g^{jk}\Phi_{jcd}\Phi_{ike}\psi^{cd}t^e t_a\\
& - g^{jk}g^{mn}\Phi_{jma}\Phi_{ikn} + \frac{1}{2}\Phi_{icd}\Pi_{be}t_a\left(\psi^{cb}\psi^{de} + \frac{1}{2}\psi^{be}t^c t^d\right)\\
& - \Phi_{icd}\Pi_{ba}t^c\left(\psi^{bd} + \frac{1}{2}t^b t^d\right) + \frac{1}{2}\gamma_2\left(t_a\psi^{cd} - 2\delta^c_a t^d\right)C_{icd}
\end{aligned}
$$

$$C_{iab} = \partial_i g_{ab} - \Phi_{iab}$$

$$C_{ijab} = 2\partial_{[i}\Phi_{j]ab}$$

$$
\begin{aligned}
\mathcal{F}_a \equiv\ & \frac{1}{2}g^i_a\psi^{bc}\partial_i\Pi_{bc} - g^{ij}\partial_i\Pi_{ja} - g^{ij}t^b\partial_i\Phi_{jba} + \frac{1}{2}t_a\psi^{bc}g^{ij}\partial_i\Phi_{jbc}\\
& + t_a g^{ij}\partial_i H_j + g^i_a\Phi_{ijb}g^{jk}\Phi_{kcd}\psi^{bd}t^c - \frac{1}{2}g^i_a\Phi_{ijb}g^{jk}\Phi_{kcd}\psi^{cd}t^b\\
& - g^i_a t^b\partial_i H_b + g^{ij}\Phi_{icd}\Phi_{jba}\psi^{bc}t^d - \frac{1}{2}t_a g^{ij}g^{mn}\Phi_{imc}\Phi_{njd}\psi^{cd}\\
& - \frac{1}{4}t_a g^{ij}\Phi_{icd}\Phi_{jbe}\psi^{cb}\psi^{de} + \frac{1}{4}t_a\Pi_{cd}\Pi_{be}\psi^{cb}\psi^{de} - g^{ij}H_i\Pi_{ja}\\
& - t^b g^{ij}\Pi_{bi}\Pi_{ja} - \frac{1}{4}g^i_a\Phi_{icd}t^c t^d\Pi_{be}\psi^{be} + \frac{1}{2}t_a\Pi_{cd}\Pi_{be}\psi^{ce}t^d t^b\\
& + g^i_a\Phi_{icd}\Pi_{be}t^c t^b\psi^{de} - g^{ij}\Phi_{iba}t^b\Pi_{je}t^e - \frac{1}{2}g^{ij}\Phi_{icd}t^c t^d\Pi_{ja}\\
& - g^{ij}H_i\Phi_{jba}t^b + g^i_a\Phi_{icd}H_b\psi^{bc}t^d + \gamma_2\left(g^{id}C_{ida} - \frac{1}{2}g^i_a\psi^{cd}C_{icd}\right)\\
& + \frac{1}{2}t_a\Pi_{cd}\psi^{cd}H_b t^b - t_a g^{ij}\Phi_{ijc}H_d\psi^{cd} + \frac{1}{2}t_a g^{ij}H_i\Phi_{jcd}\psi^{cd}\\
& - 16\pi t^a T_{ab}
\end{aligned}
$$

Constraint equations

$$H_a \equiv g_{ab}\partial^c\partial_c x^b$$

$a, b, \ldots =$ spacetime indices t, x, y, z

$i, j, \ldots =$ spatial indices x, y, z

$\alpha, \beta, \ldots =$ equation indices $g_{ab}, \Pi_{ab}, \Phi_{iab}$

Sum over repeated indices

$G = c = 1$

Lindblom+ Class. Quant. Grav. **23**, S447 (2006)

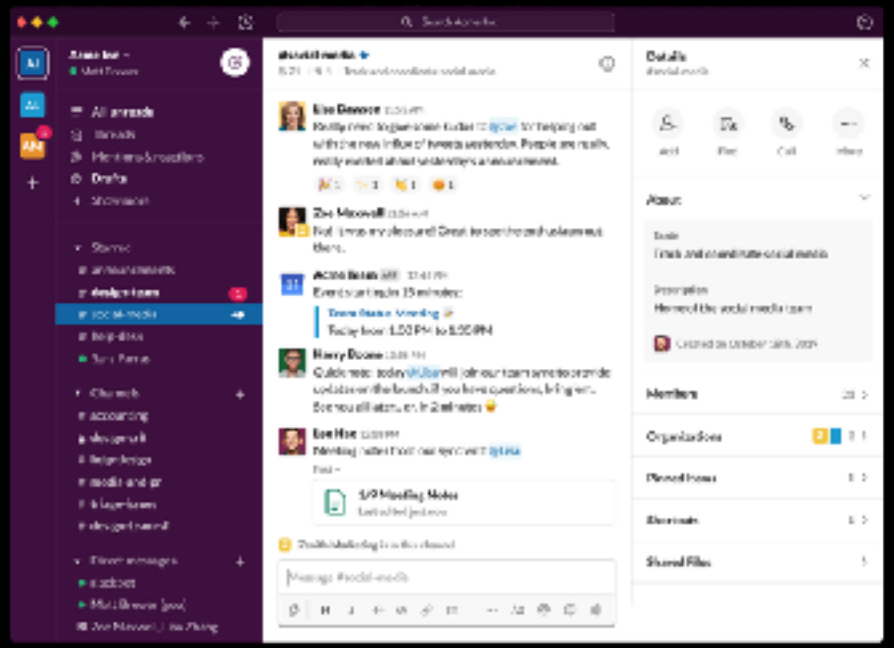
SpECTRE

- Open, next-gen. NR code
  - Discontinuous Galerkin (DG)
  - Task-based parallelism

SpEC	SpECTRE
Home-grown	charm++ charm.cs.illinois.edu
Cores run same code on different parts of grid	Cores ask scheduler for tasks from queue

Many sync points

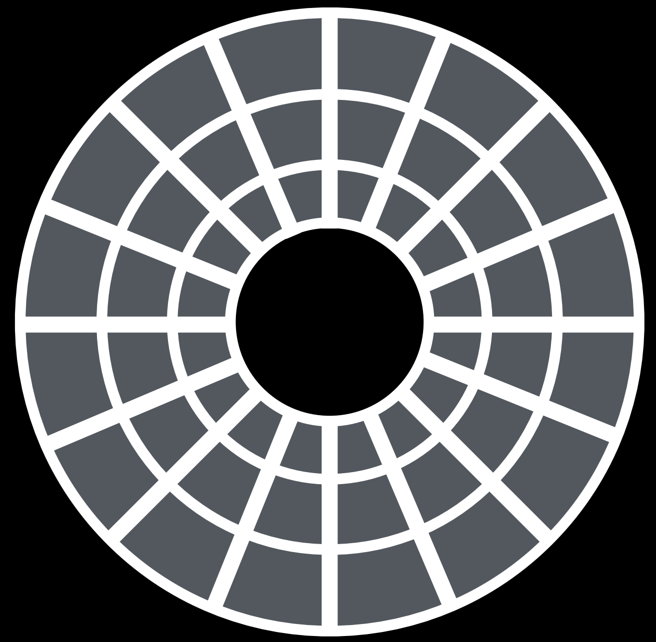
Scales to 50 cores



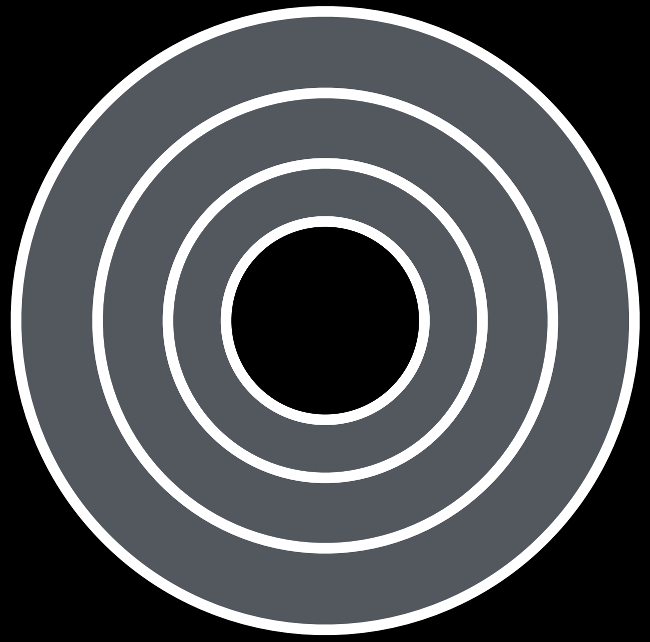
Few sync points

Scales to 100k cores

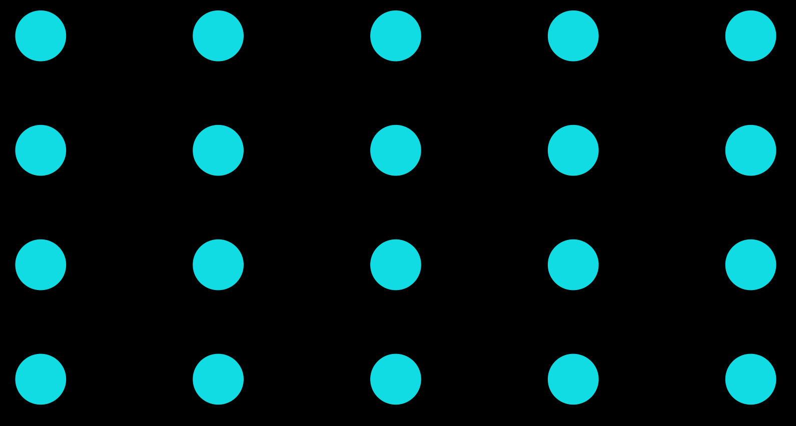
DG



Pseudospectral



Finite Diff.

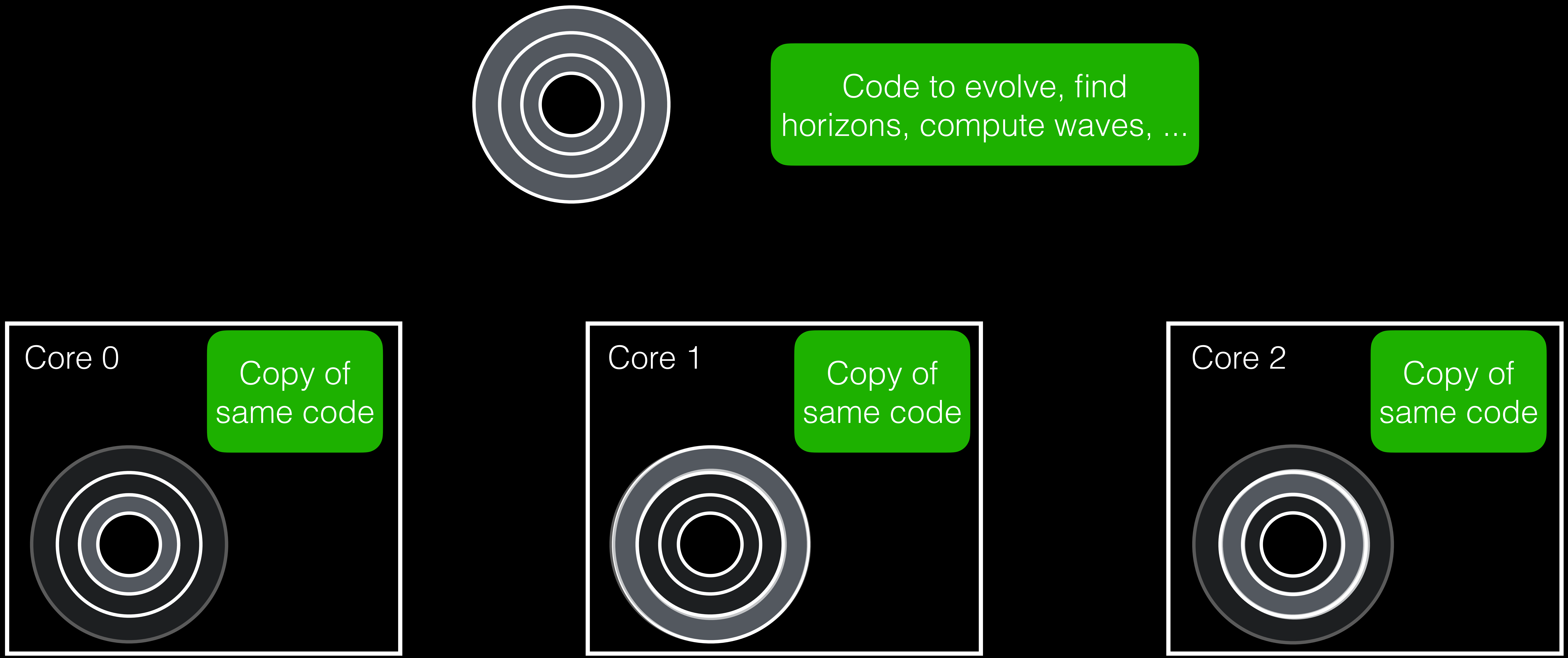


$$f(x) = \sum_{n=0}^{N} a_n \phi(x)$$

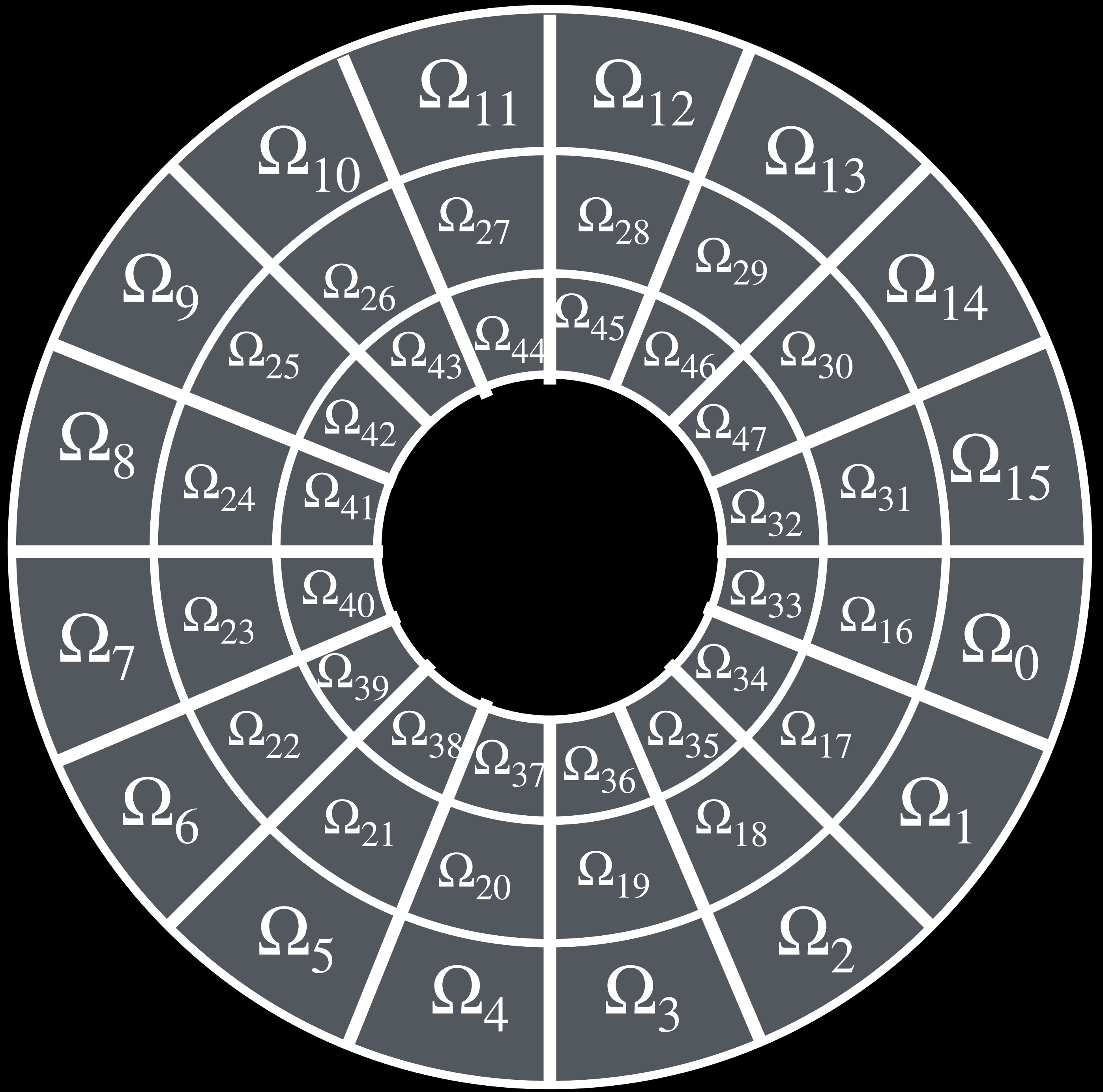
DG	Pseudospectral	Finite Diff.
Smaller N more cells	Bigger N fewer cells	Values at grid points
		Shocks
Exponential convergence *when solution smooth*		Polynomial convergence
Analytic high-order derivatives		Wide stencils *High communication on many CPUs*

Images courtesy zoom.us, slack.com

Data parallelism



Task parallelism



- = array parallel component
- = singleton parallel component
- = global cache

Parallel component

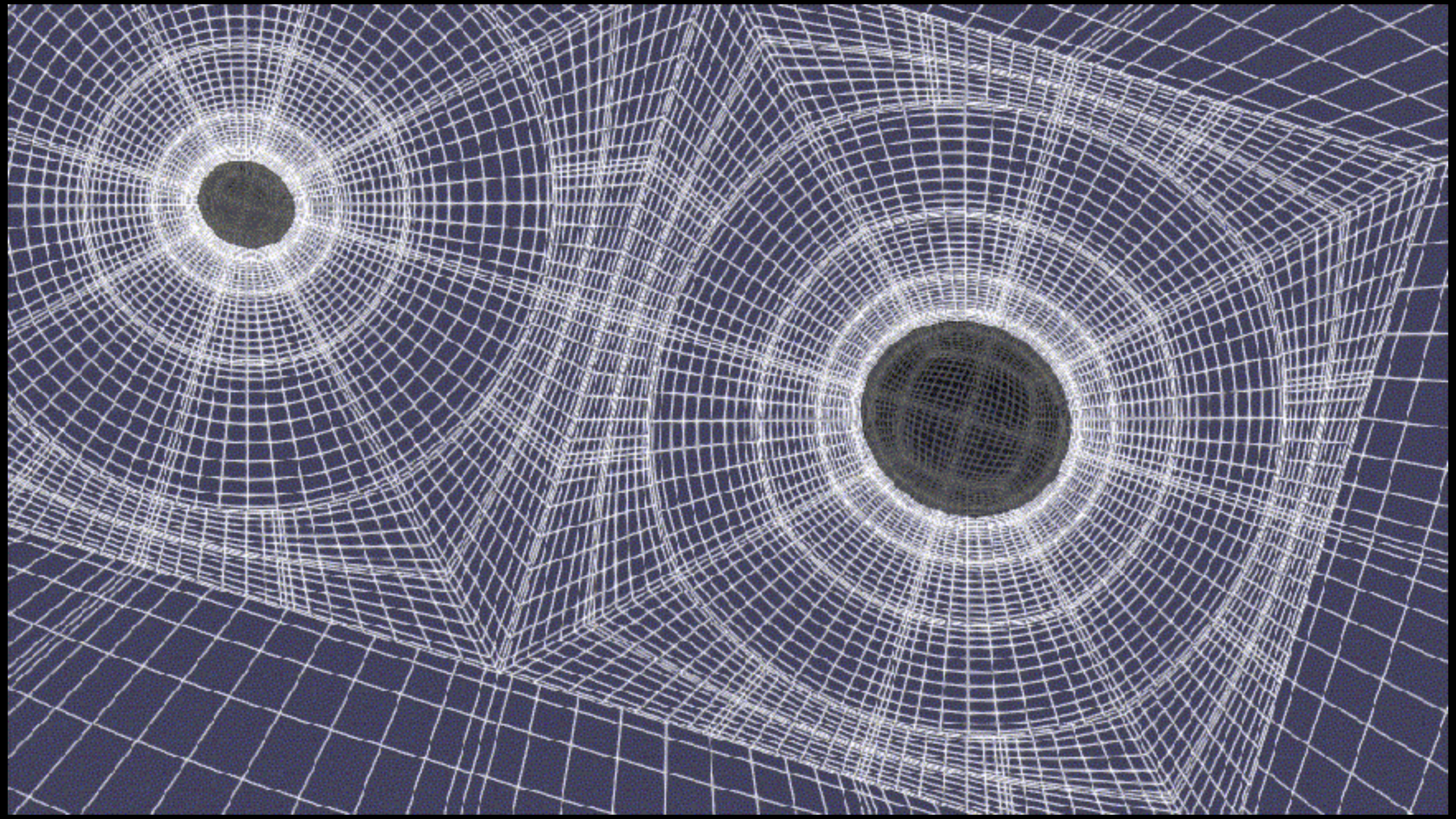
= "actor" that knows things, does things

= "distributed object"

= charm++ chare

Moving mesh

- Deform, move mesh with grid velocity
  - Track black holes, ensuring singularities remain excised, horizon exteriors not excised



Animation courtesy Kyle Nelli

Day 3

- Gravitational-wave concepts (with Dr. Jocelyn Read)
- Special guest: Haroon Khan (NASA)
- Choose one head-on collision on binary black holes and start the calculation

Two kinds of time travel

- Travel from the present to…

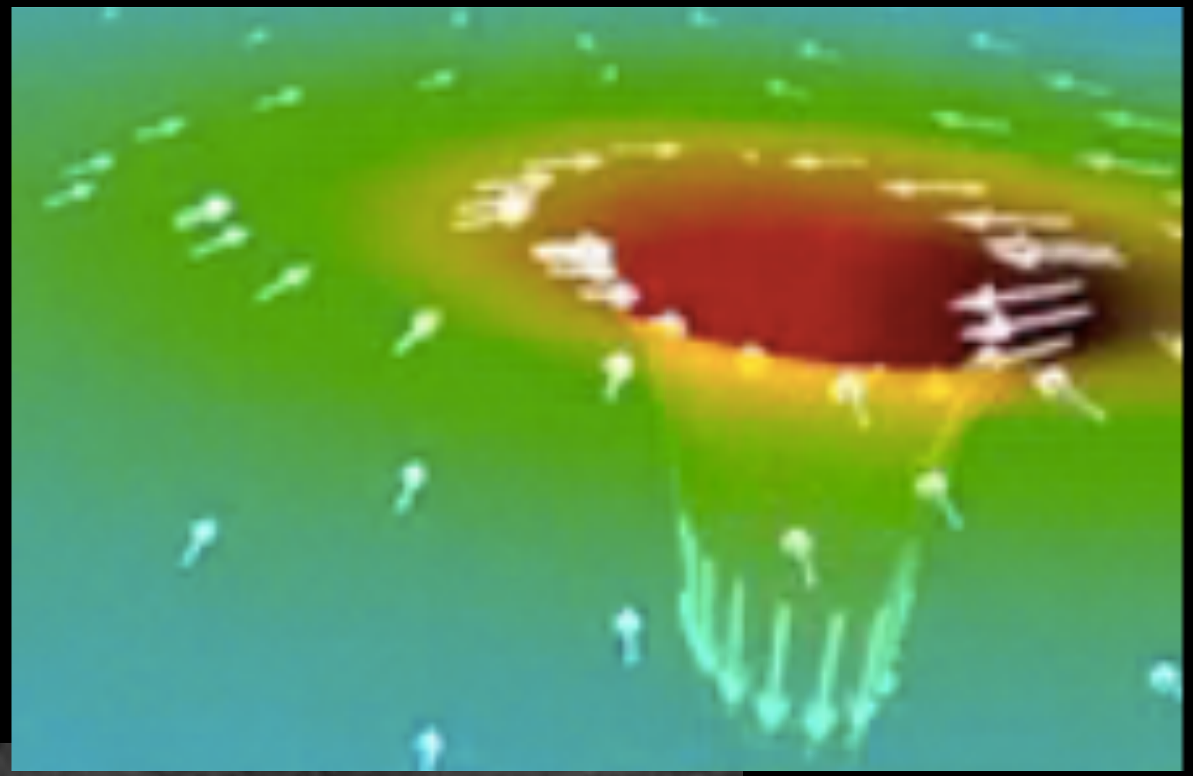
The future

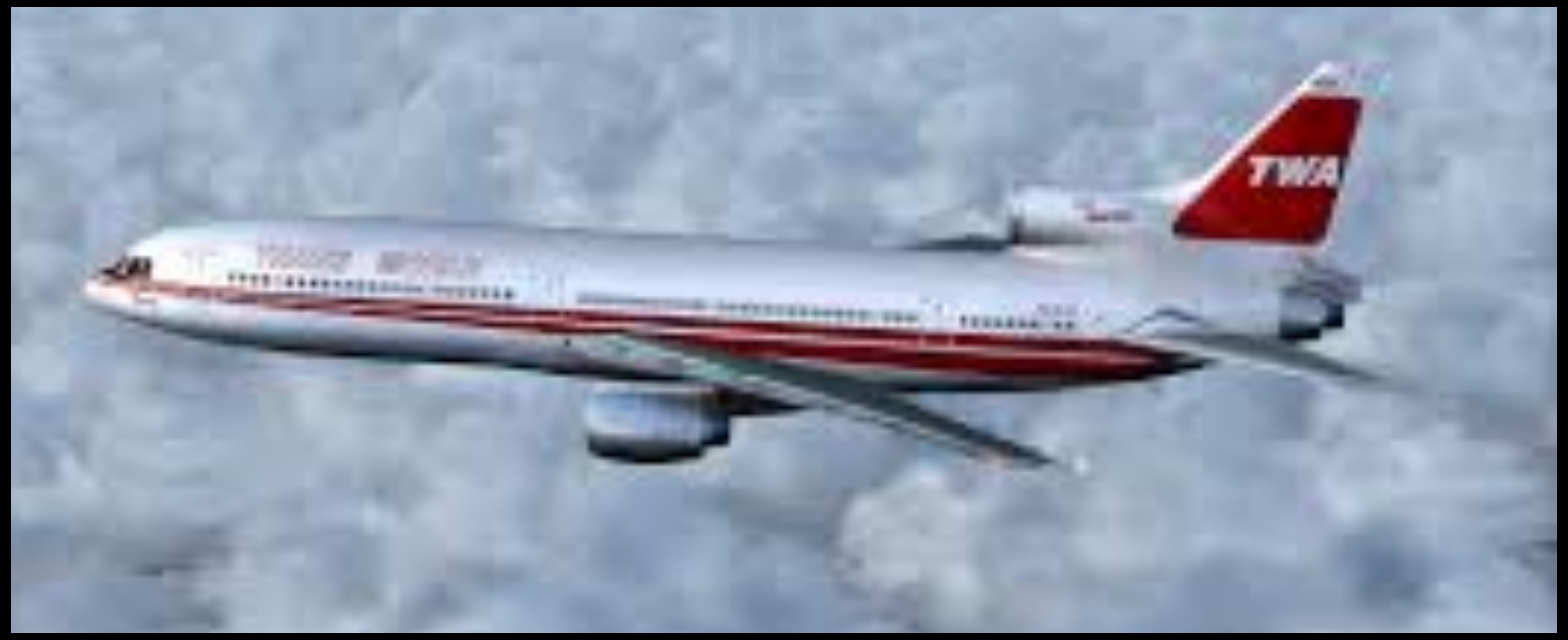
The past

Images courtesy backtothefuture.wikia.com, oocities, “Back to the Future”

Forward time travel

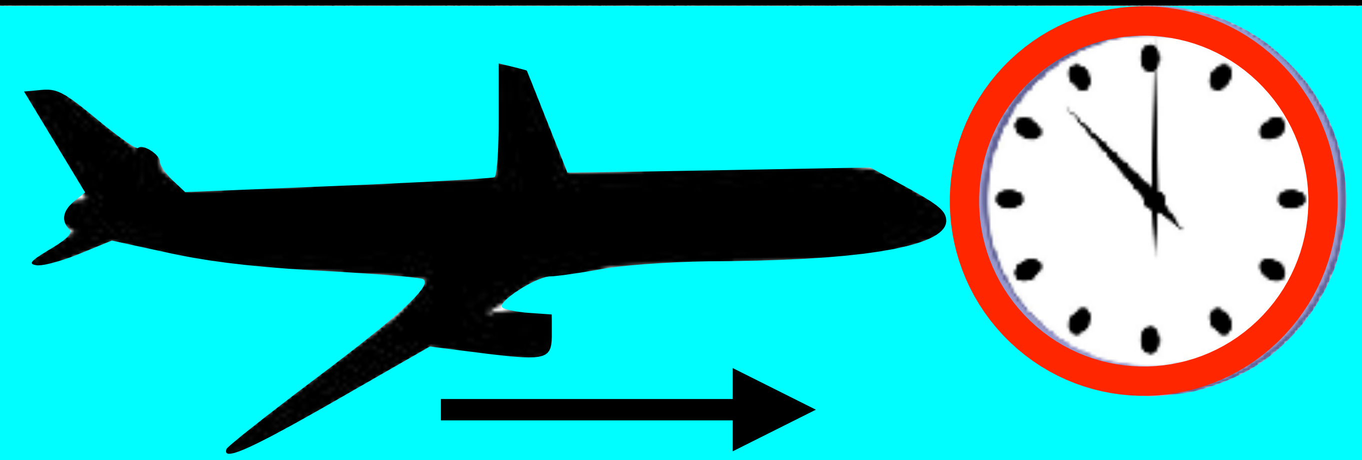
- Make your time flow slower
  - –Move closer to massive object
  - –Move faster
- Hafele & Keating 1971
  - –Fly plane clock around world
  - –Compare with ground clock before, after flight





Images courtesy SXS Collaboration, wikipedia

Forward time travel in 1971



Theory of relativity predicts…

Farther from Earth: ticks faster vs. ground

144±14 ns *more* than ground

Moves faster: ticks slower vs. ground

184±18 ns *less* than ground

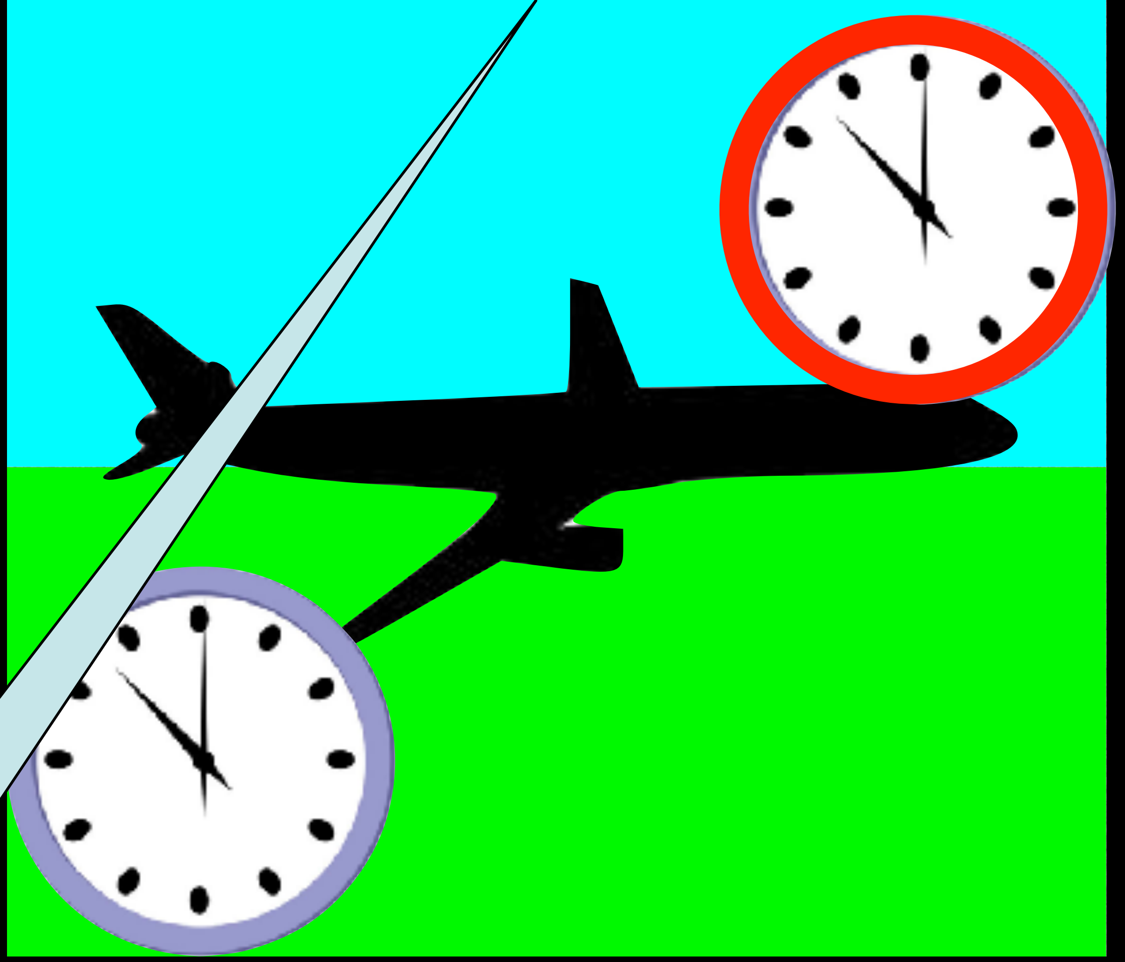
Bottome line:

40±23 ns *less* than ground

1 ns = 1 billionth of a second (ns)

Result: compare clocks after flight around the world

59±10 ns less than ground



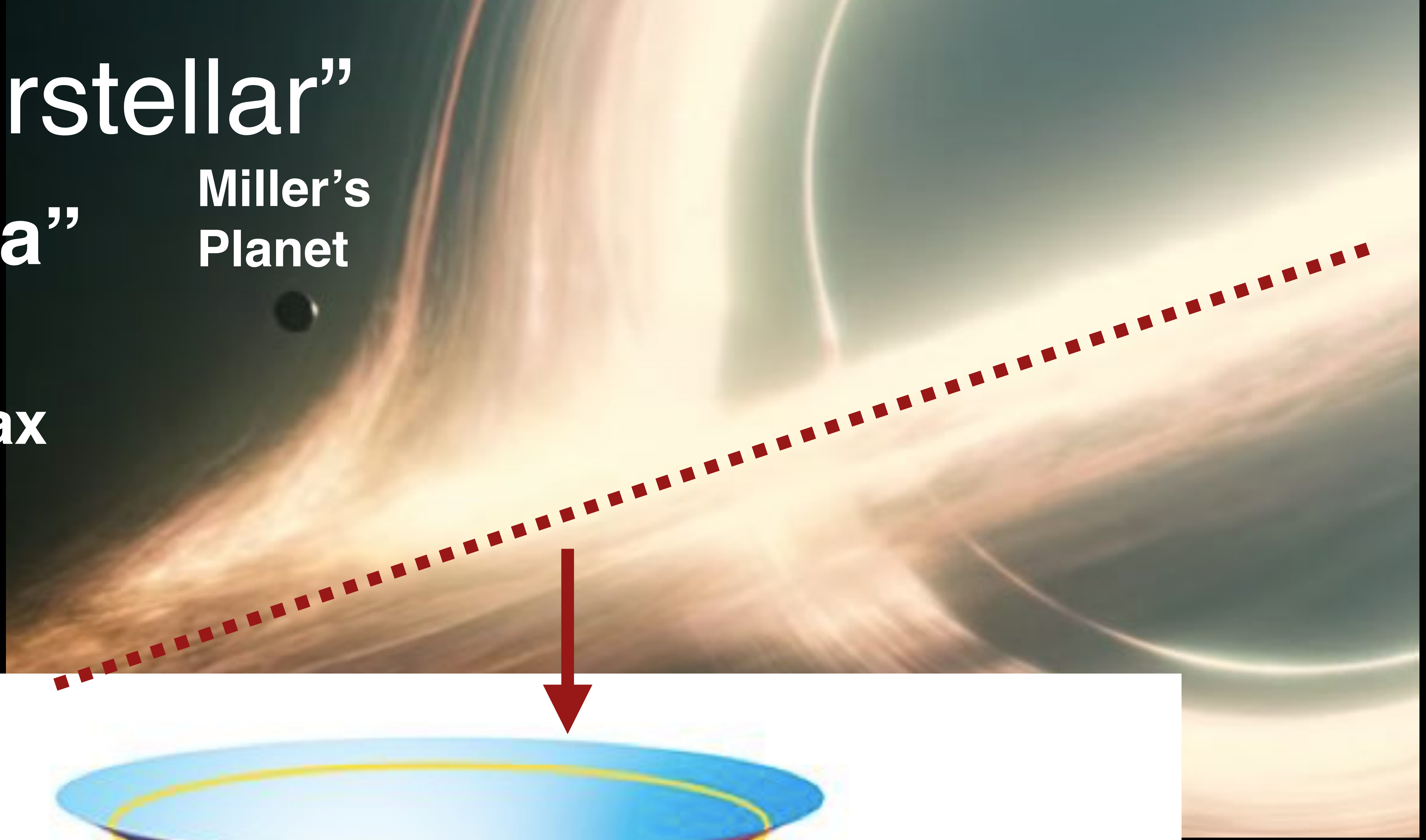
Clock & passengers went 59±10 billionths of a second into the future!

Time travel in "Interstellar"

Black hole "Gargantua"

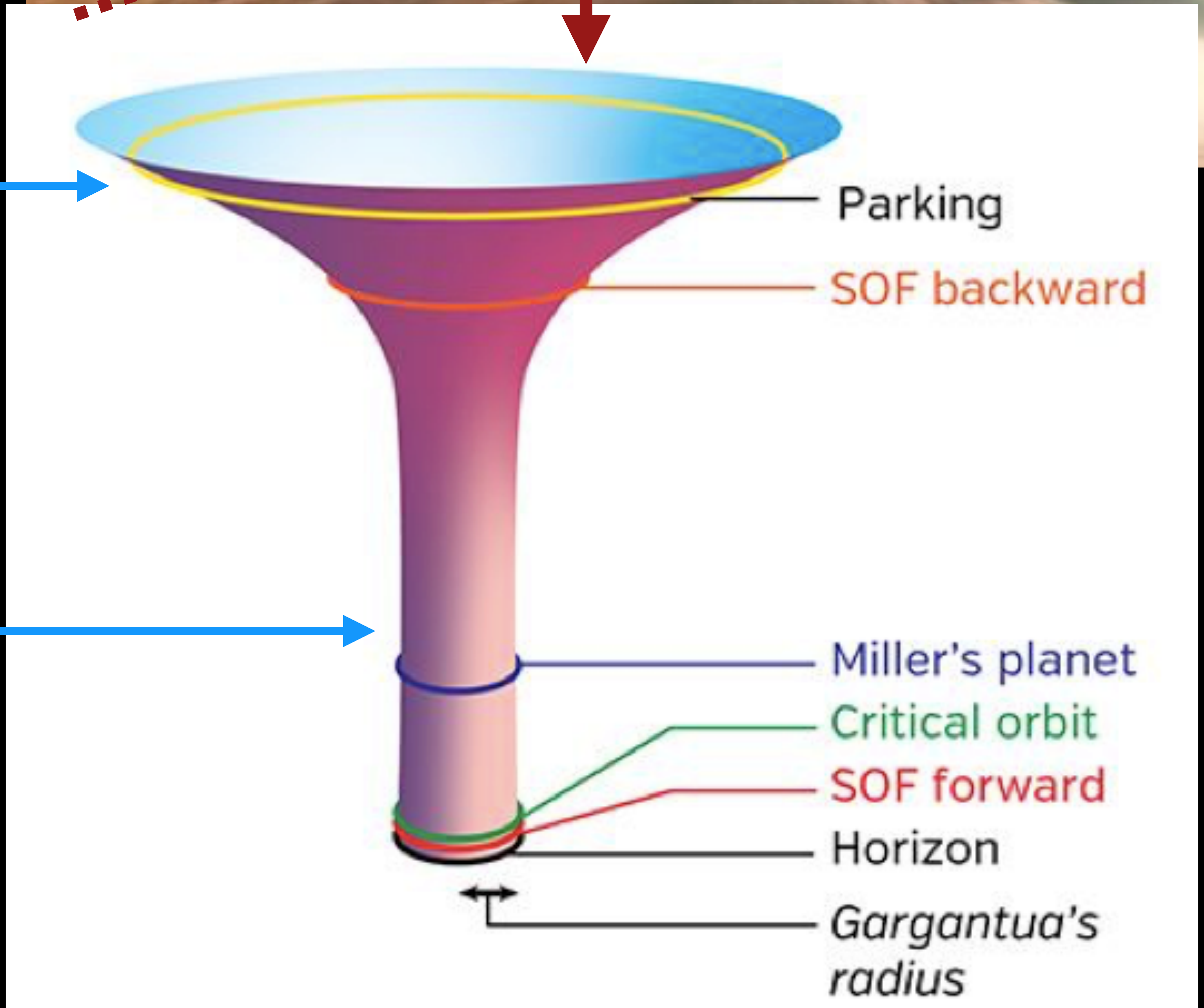
Mass: 100 000 000 ☼

Spin: 99.999999999999% max



7 years

1 hour



Images courtesy Kip Thorne, Paramount

GPS

- How does GPS work?

“It’s 4:59:58 PM”

“It’s 4:59:58 PM”

“It’s 4:59:58 PM”

“It’s 4:59:58 PM”

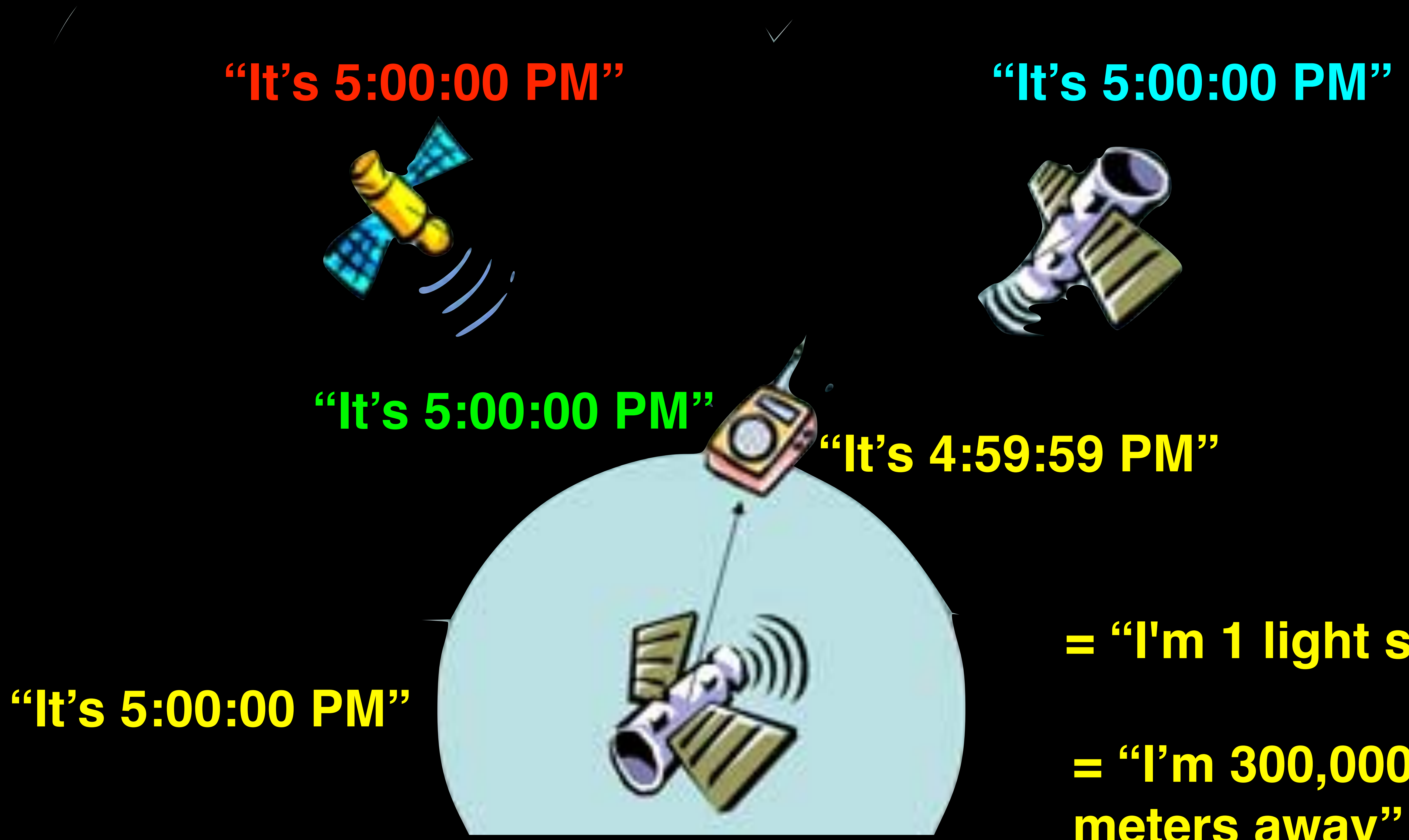
Images courtesy NOAA, how-gps-works.com

GPS

- How does GPS work?

GPS

- How does GPS work?



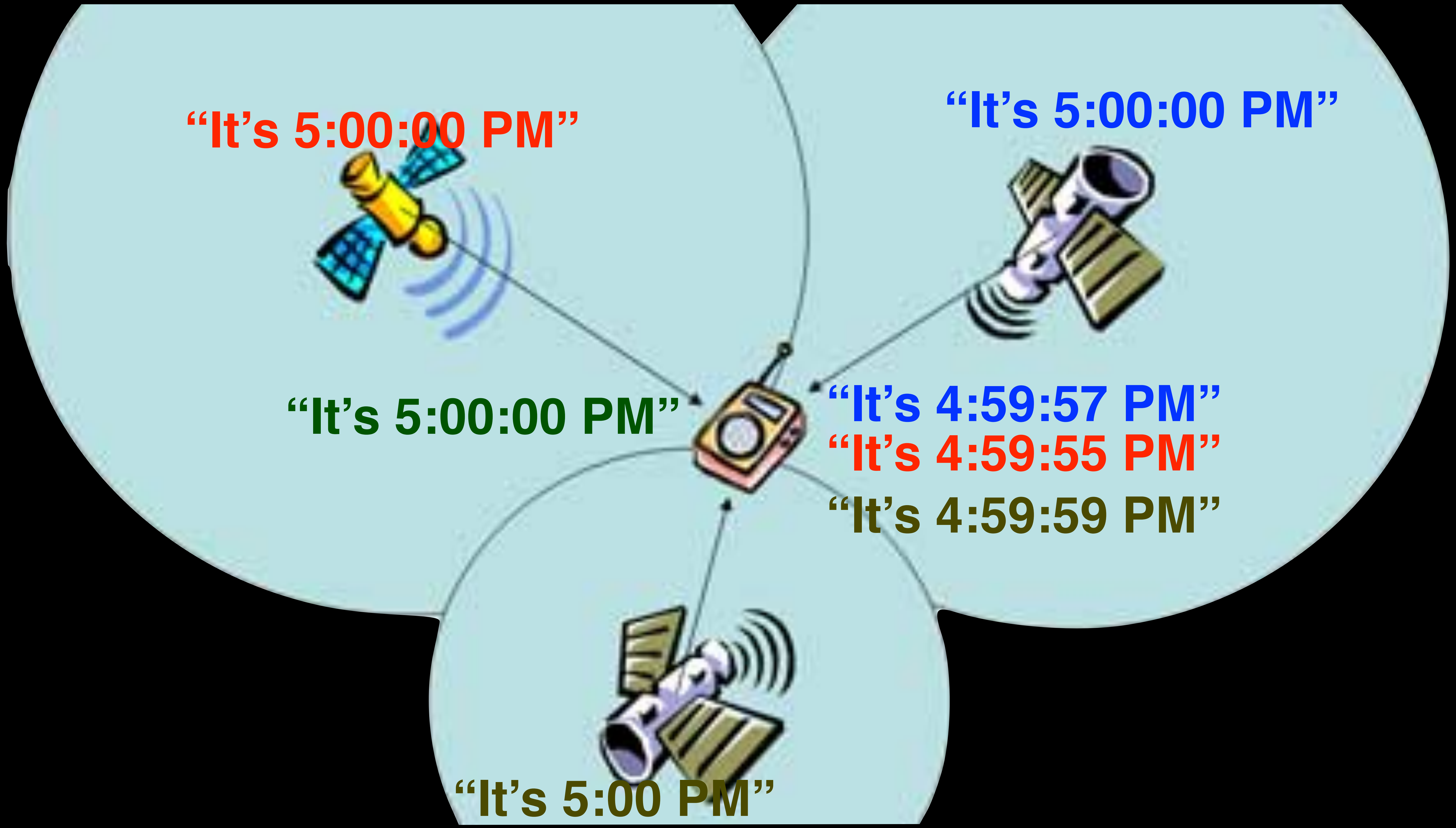
= “I'm 1 light second away”

= “I’m 300,000,000 meters away”

Images courtesy NOAA, how-gps-works.com

GPS

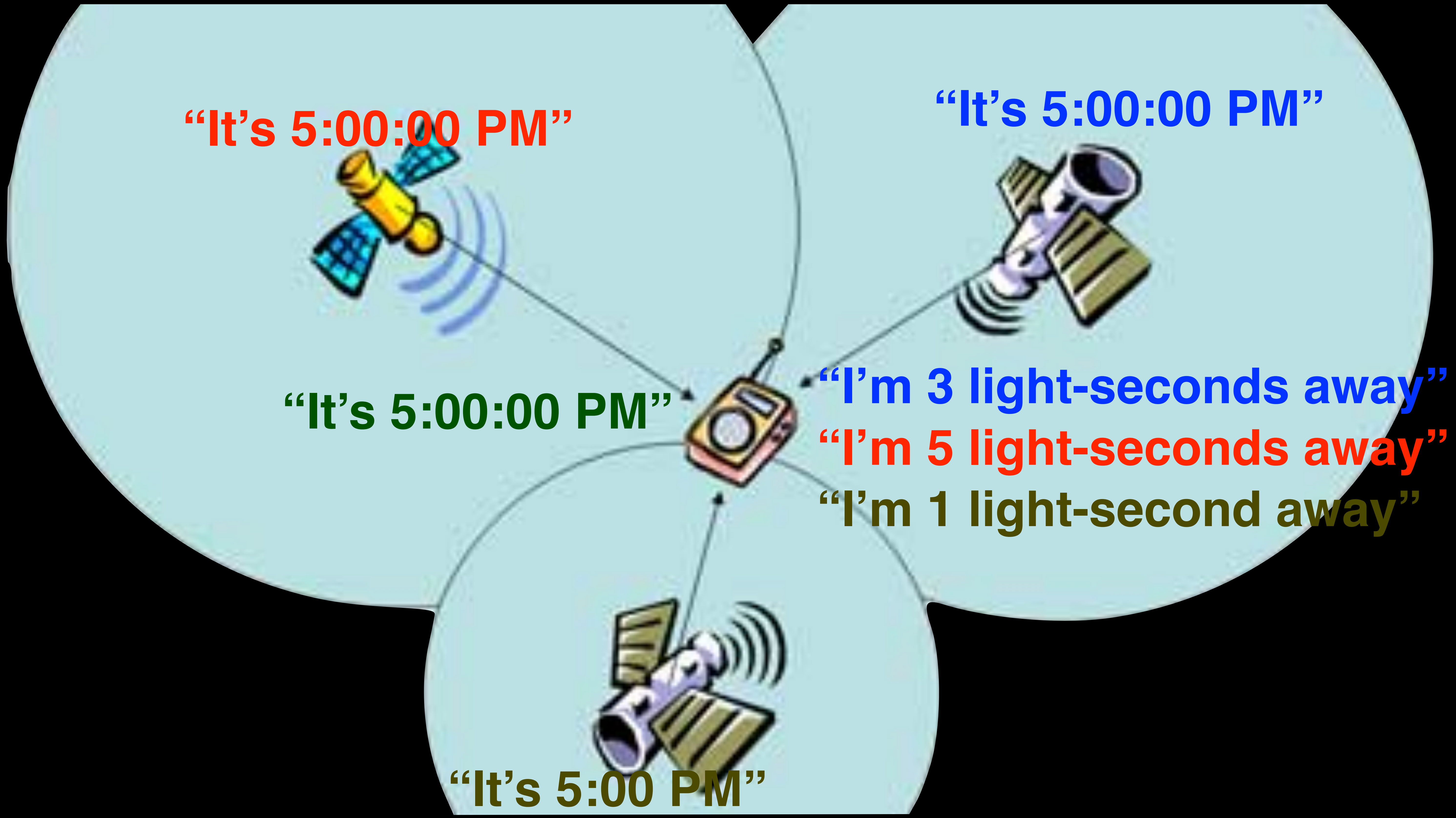
- How does GPS work?



Images courtesy NOAA, how-gps-works.com

GPS

- How does GPS work?



Images courtesy NOAA, how-gps-works.com

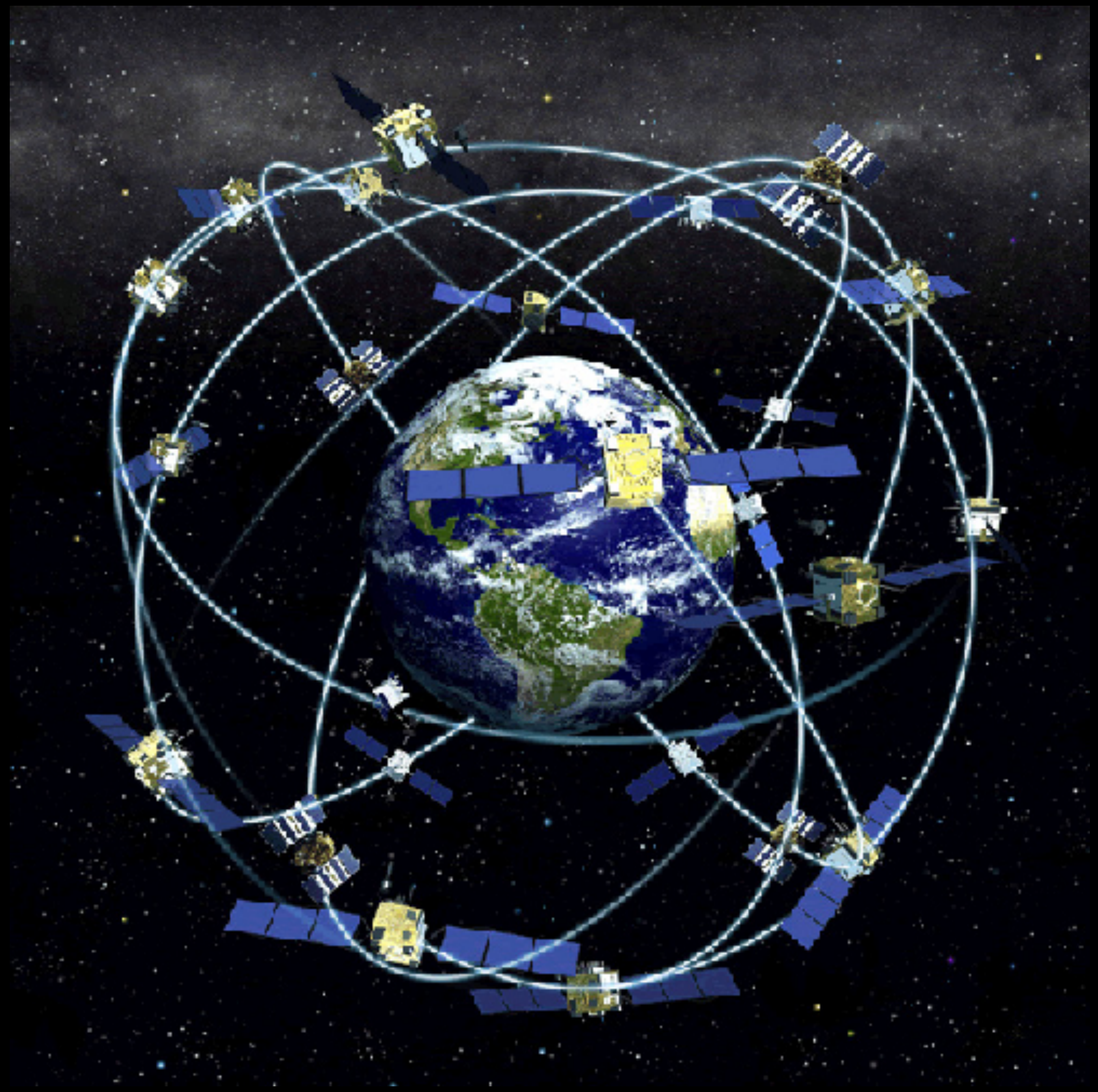
GPS

- How does GPS work?

Images courtesy NOAA, how-gps-works.com

GPS & forward time travel

- GPS must account for both "time travel" effects



Goal: position accuracy of about 2 m

Light travels 2 m in about 7 ns

So clocks really give time to ns precision:
"It's 4:59:59.123456789 PM"

That's no problem for atomic clocks, but…

Satellite clocks are higher, moving: tick differently!

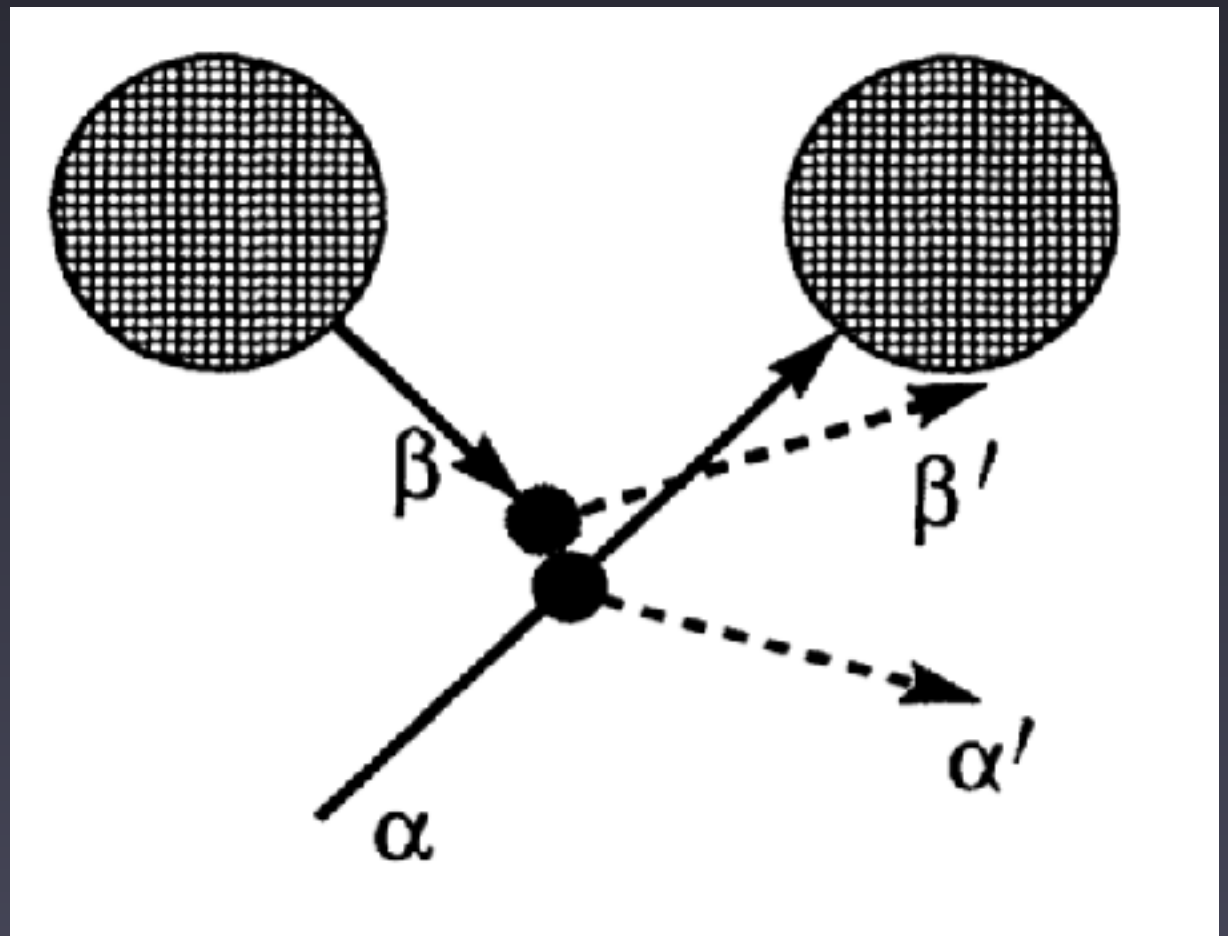
Ignore this, and errors start to build up, exceeding 2 m in less than a minute

Images courtesy NOAA, how-gps-works.com

Backward time travel

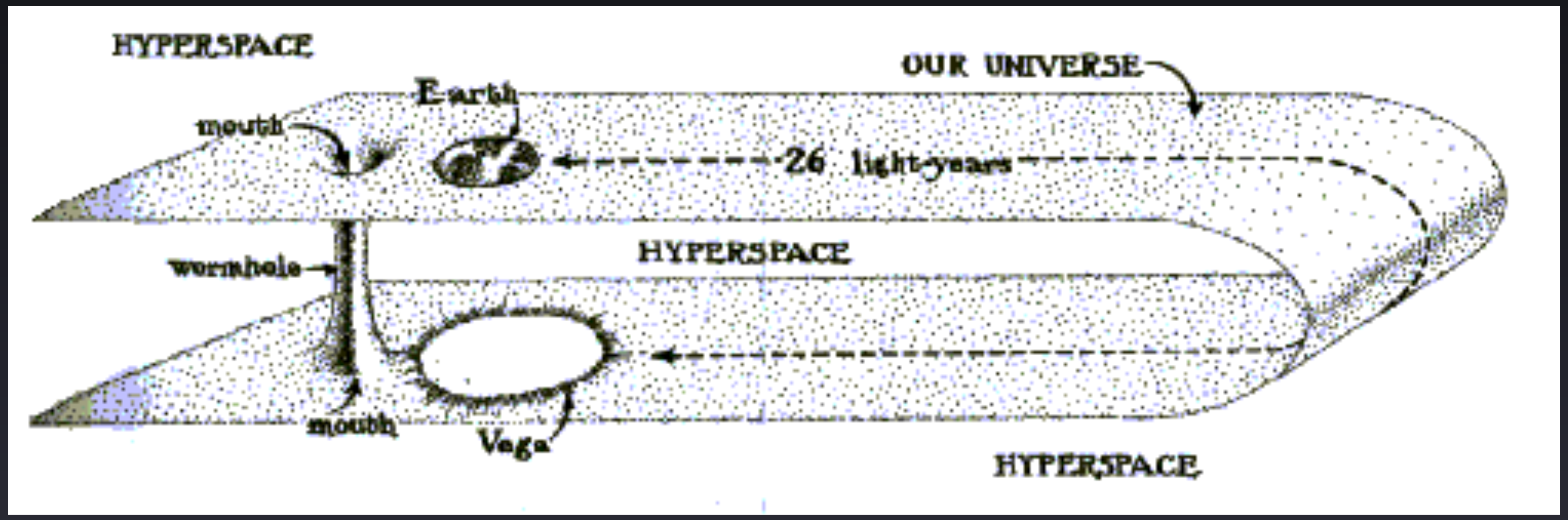
- "Matricide paradox"
  - Go back in time and prevent yourself from being born?
- "Billiard ball paradox" (Polchinski, 1988)
  - Can ball go back in time & collide with itself, preventing itself from going back in time?
  - Echeverria, Klinkhammer, and Thorne [EKT], 1991

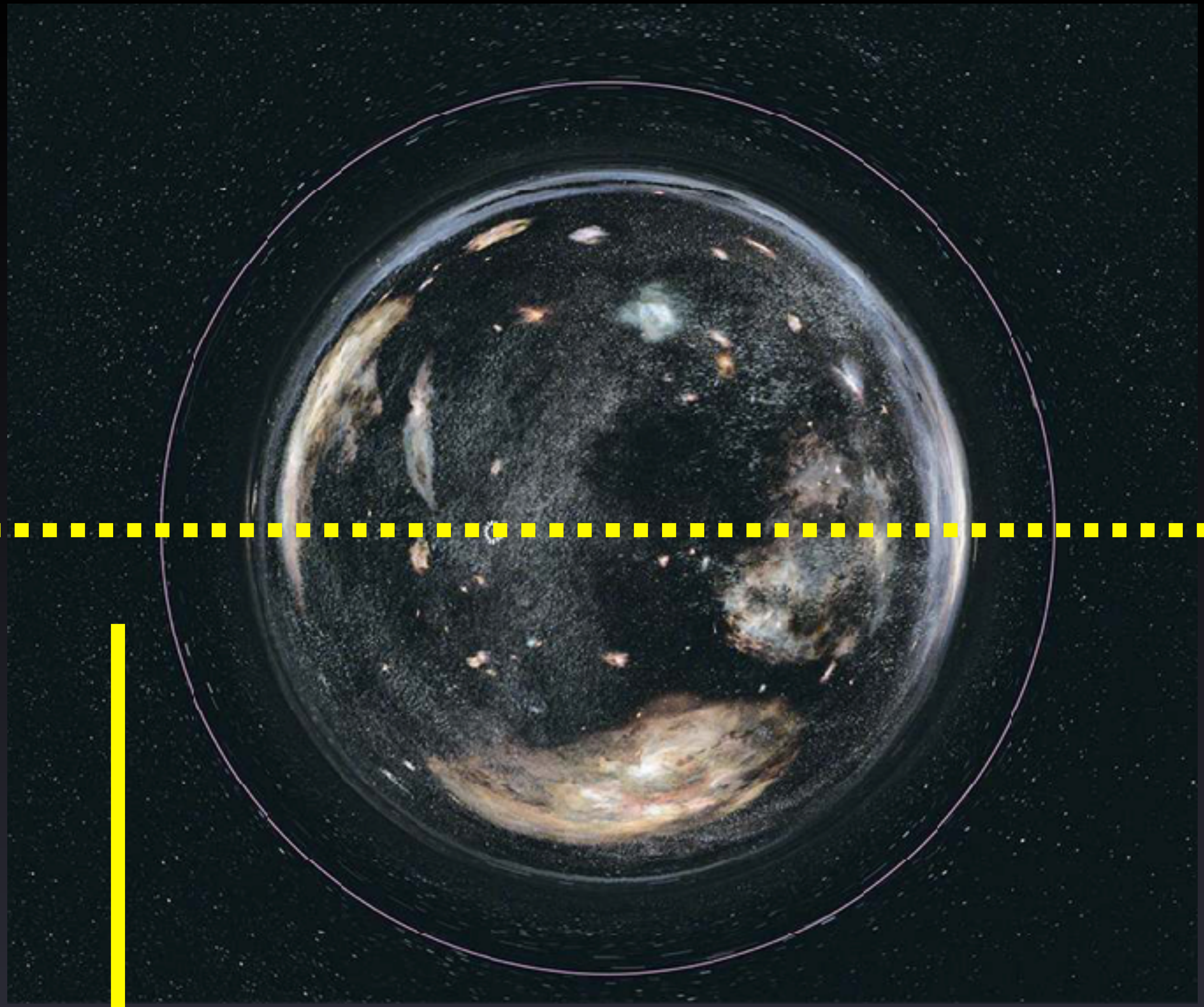
Image courtesy wikipedia



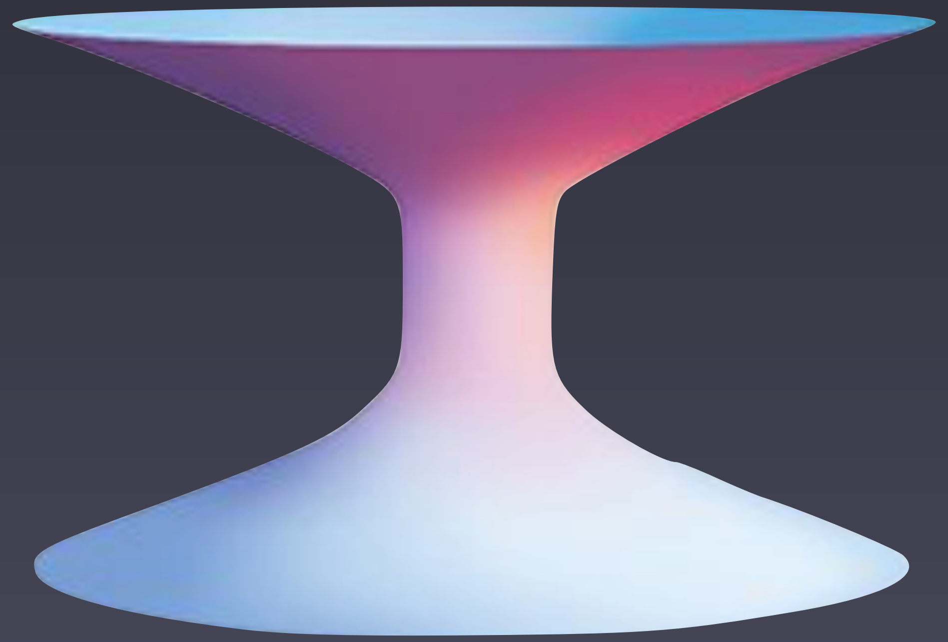
Wormhole in “Interstellar”

- Connect distant parts of universe



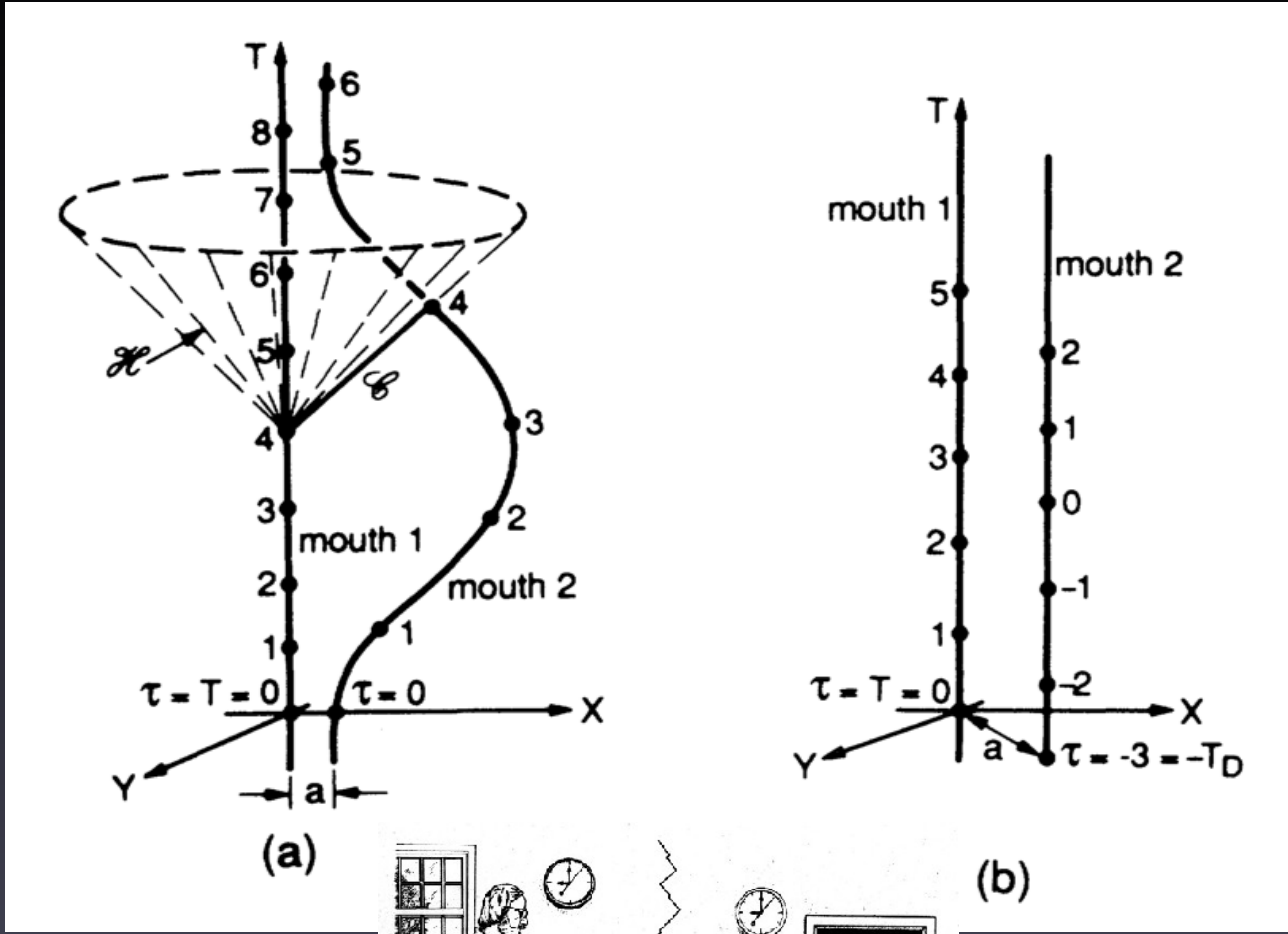


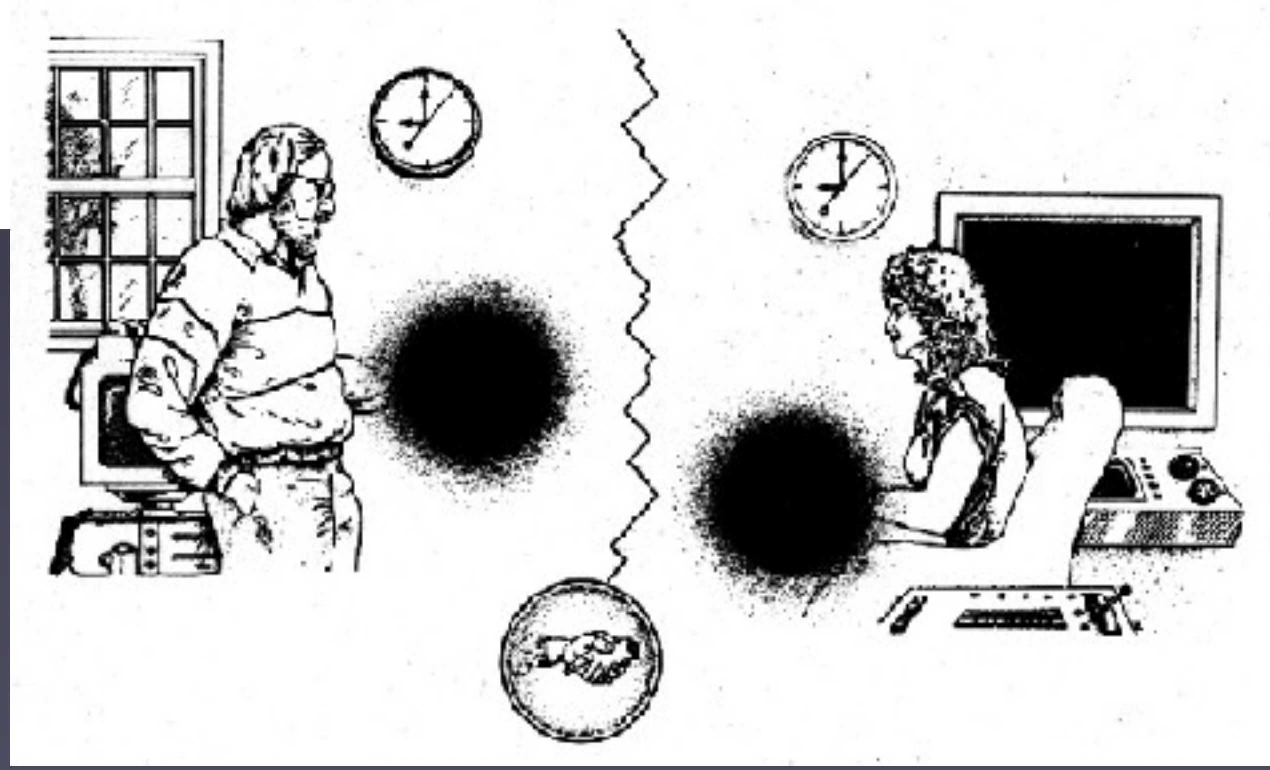
- Wormholes probably can’t exist
  - Require “negative mass” to avoid collapsing
- Can be used to make a time machine



Images courtesy Kip Thorne, Paramount, NASA

“Time machine” spacetimes



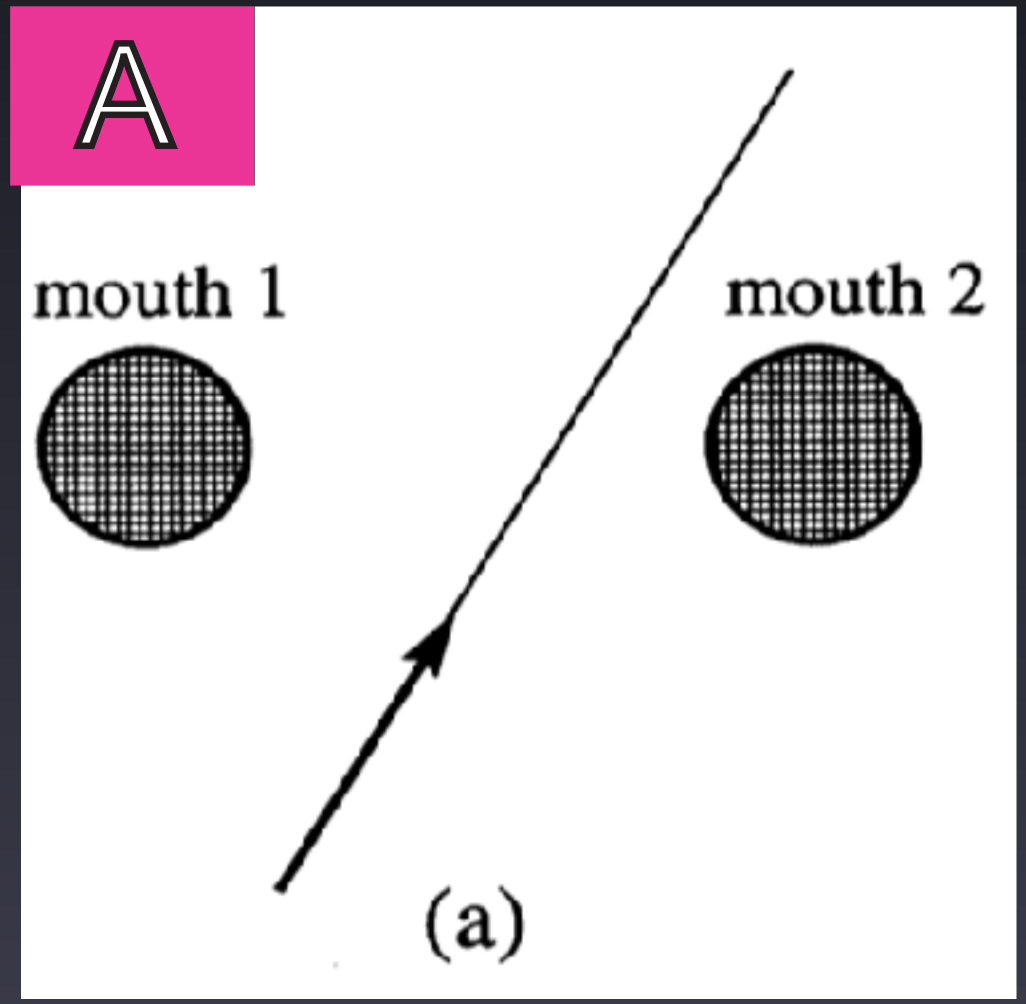


Images courtesy Kip Thorne

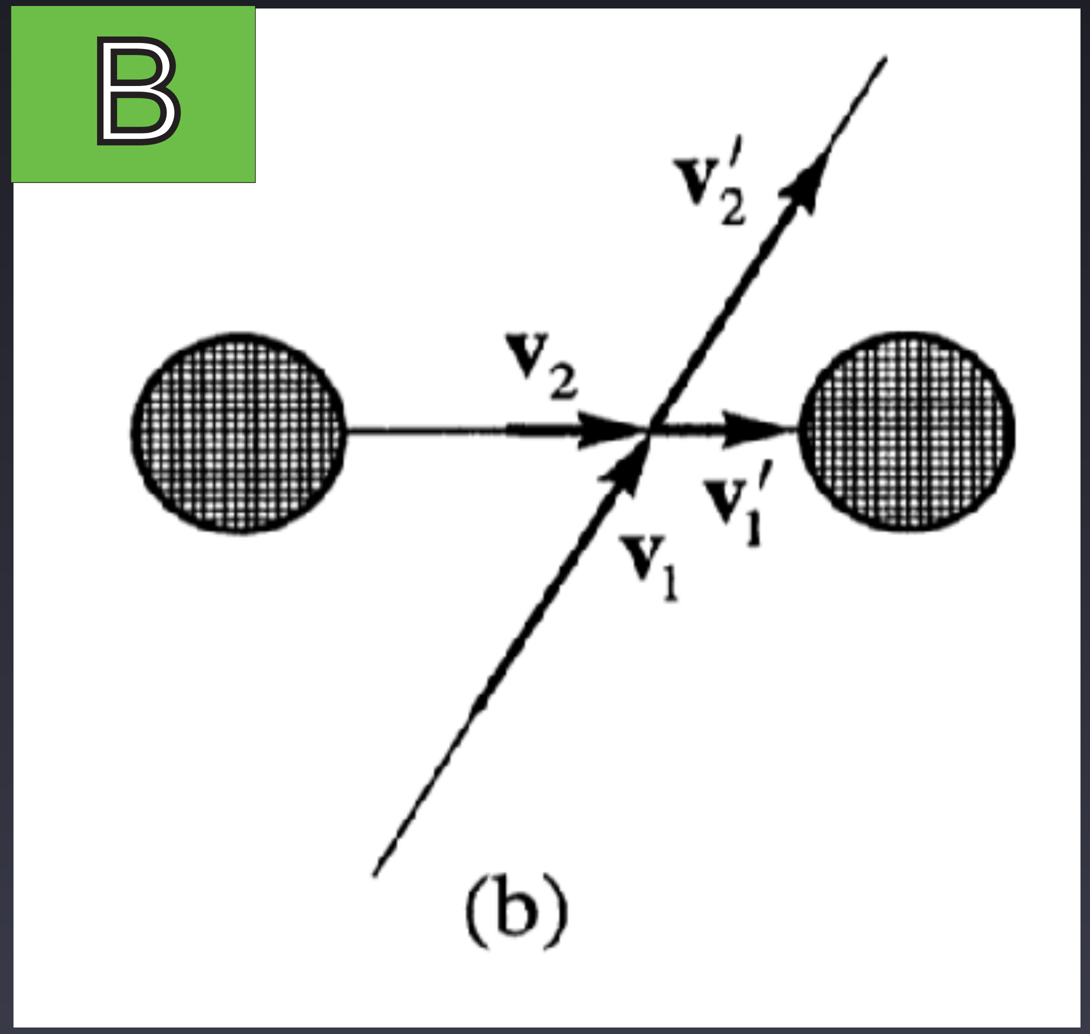
What do you think?

- A billiard ball begins with initial velocity velocity $\mathbf{v}_1$, heading between the two mouths of the time machine. Aside from the time machine, Newton's laws of motion apply. What happens?

A



B



C

Can't say: both A and B satisfy Newton's laws of motion

```
cd $HOME
cd StudentFolders
cd YOURNAME # replace YOURNAME with the name of your folder
```

```
cd $HOME
cd StudentFolders
cd YOURNAME # replace YOURNAME with the name of your folder
mkdir BlackHoleMerger
cd BlackHoleMerger
```

```
cd $HOME
cd StudentFolders
cd YOURNAME # replace YOURNAME with the name of your folder
mkdir BlackHoleMerger
cd BlackHoleMerger
source $HOME/spec/MakefileRules/this_machine.env
```

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cd BlackHoleMerger
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PrepareID -t bbh2 -no-reduce-ecc
```

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cd BlackHoleMerger
source $HOME/spec/MakefileRules/this_machine.env
PrepareID -t bbh2 -no-reduce-ecc
nano Params.input
Omega0 = 0.0;
adot0 = 0.0;
D0 = 35.0;
MassRatio = 1.2; #or 1.0, or something in between
@SpinA = (0.0, 0.0, 0.0); #can make 1 component up to 0.2 instead of 0.1
@SpinB = (0.0, 0.0, 0.0);
```

```
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# @SpinB = (0.0, 0.0, 0.0)
nano Ev/DoMultipleRuns.input
# my MaxLev = 1
```

```
cd $HOME
cd StudentFolders
cd YOURNAME # replace YOURNAME with the name of your folder
mkdir BlackHoleMerger
cd BlackHoleMerger
source $HOME/spec/MakefileRules/this_machine.env
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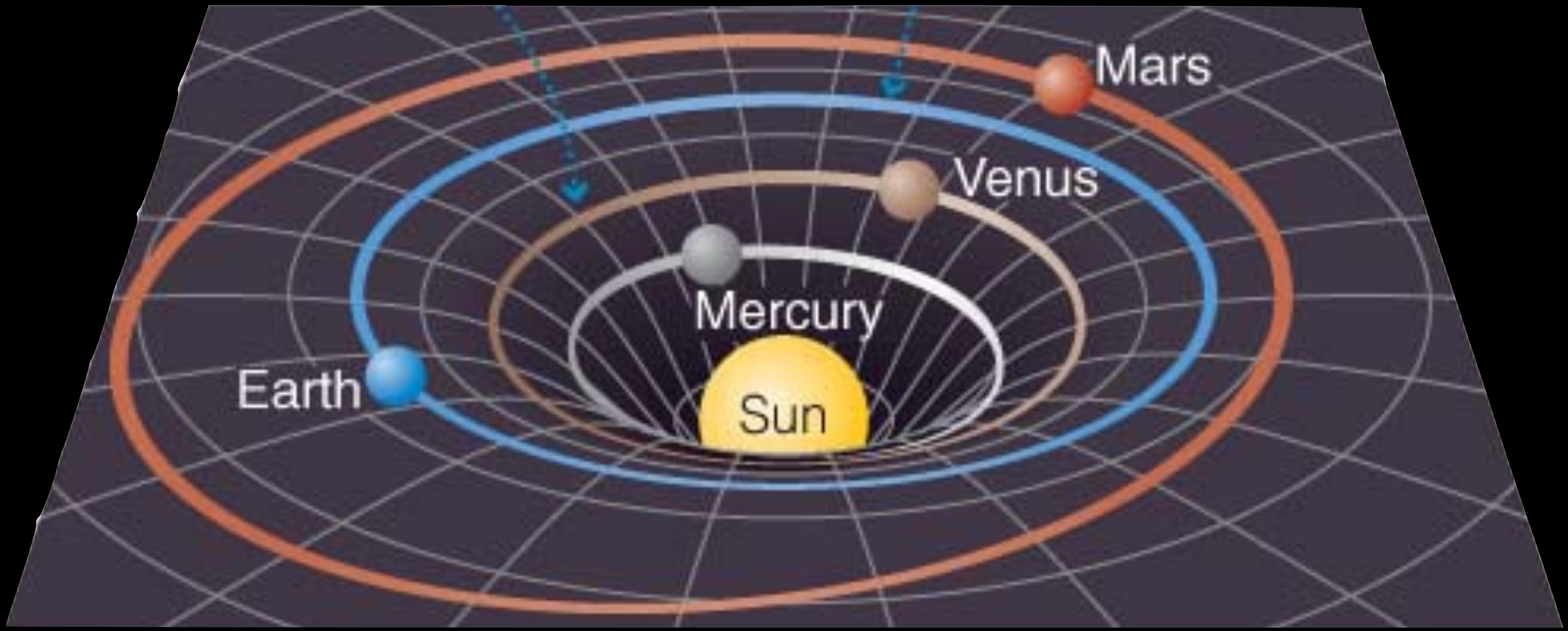
of 0.1
# @SpinB = (0.0, 0.0, 0.0)
nano Ev/DoMultipleRuns.input
# my MaxLev = 1
./StartJob.sh
```

```
squeue
scontrol show jobid -dd YOUR_JOB_ID
ShowQueue
```

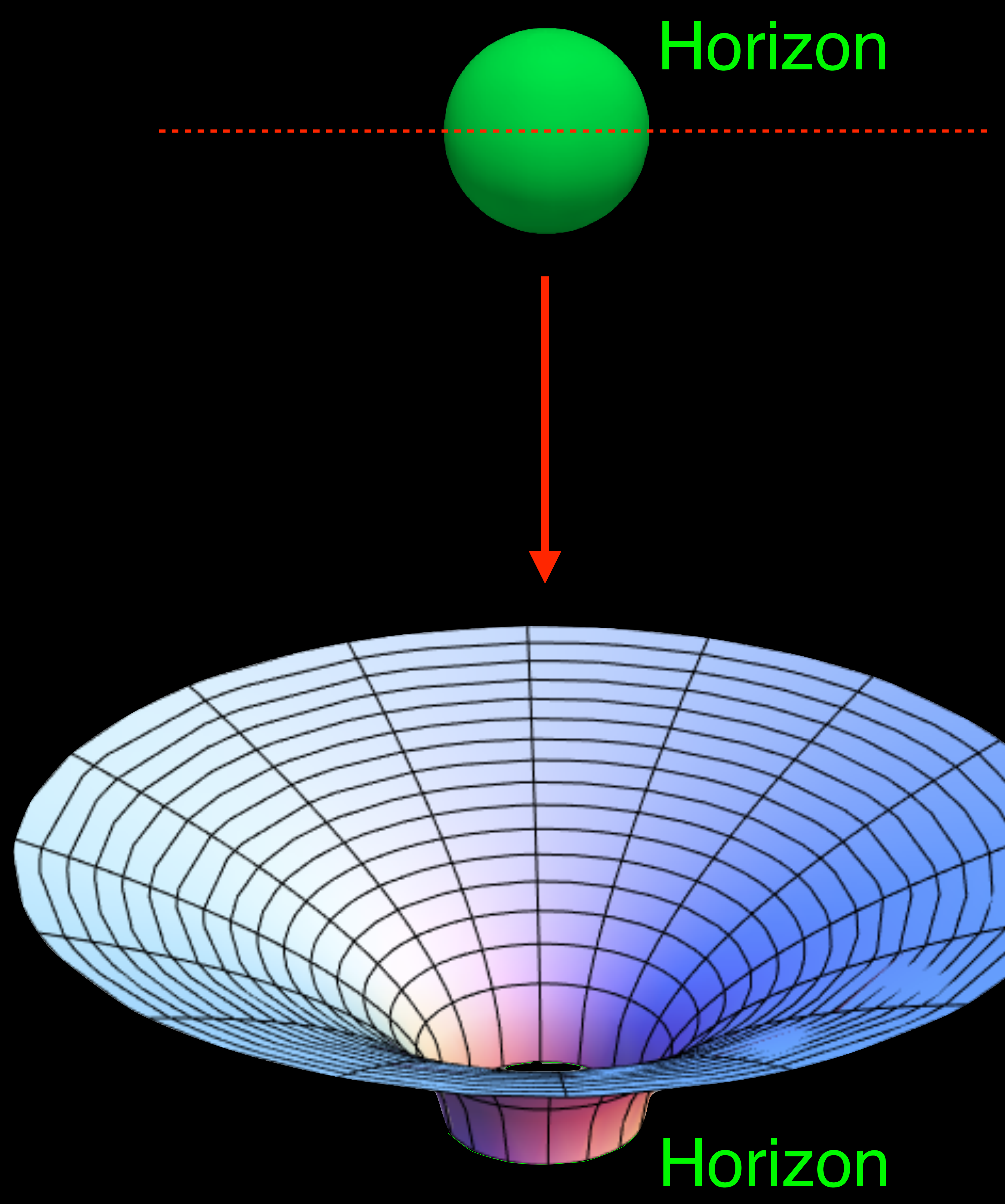
Curved spacetime

"Matter tells spacetime how to curve and space-time tells matter how to move."
- John Wheeler



Extremely curved spacetime: black holes

- Gravity so strong...
  - Nothing (even light) can escape from inside hole's horizon (surface)
  - Singularity inside horizon: *infinitely* strong gravity
- Formed when the most massive stars die



What is spacetime?

- 3 dimensions of space + 1 dimension of time
- Event = a specific place at a specific time

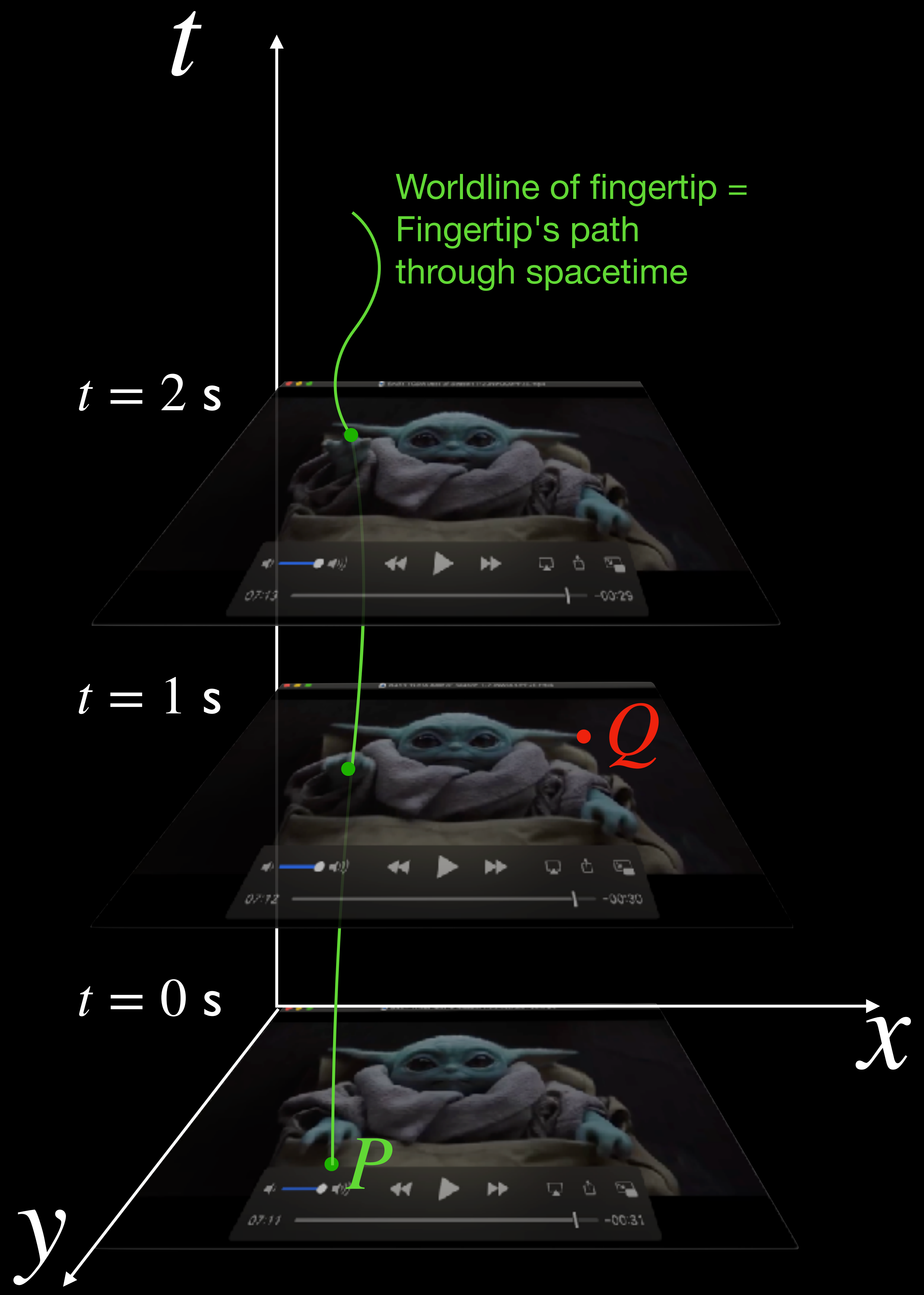
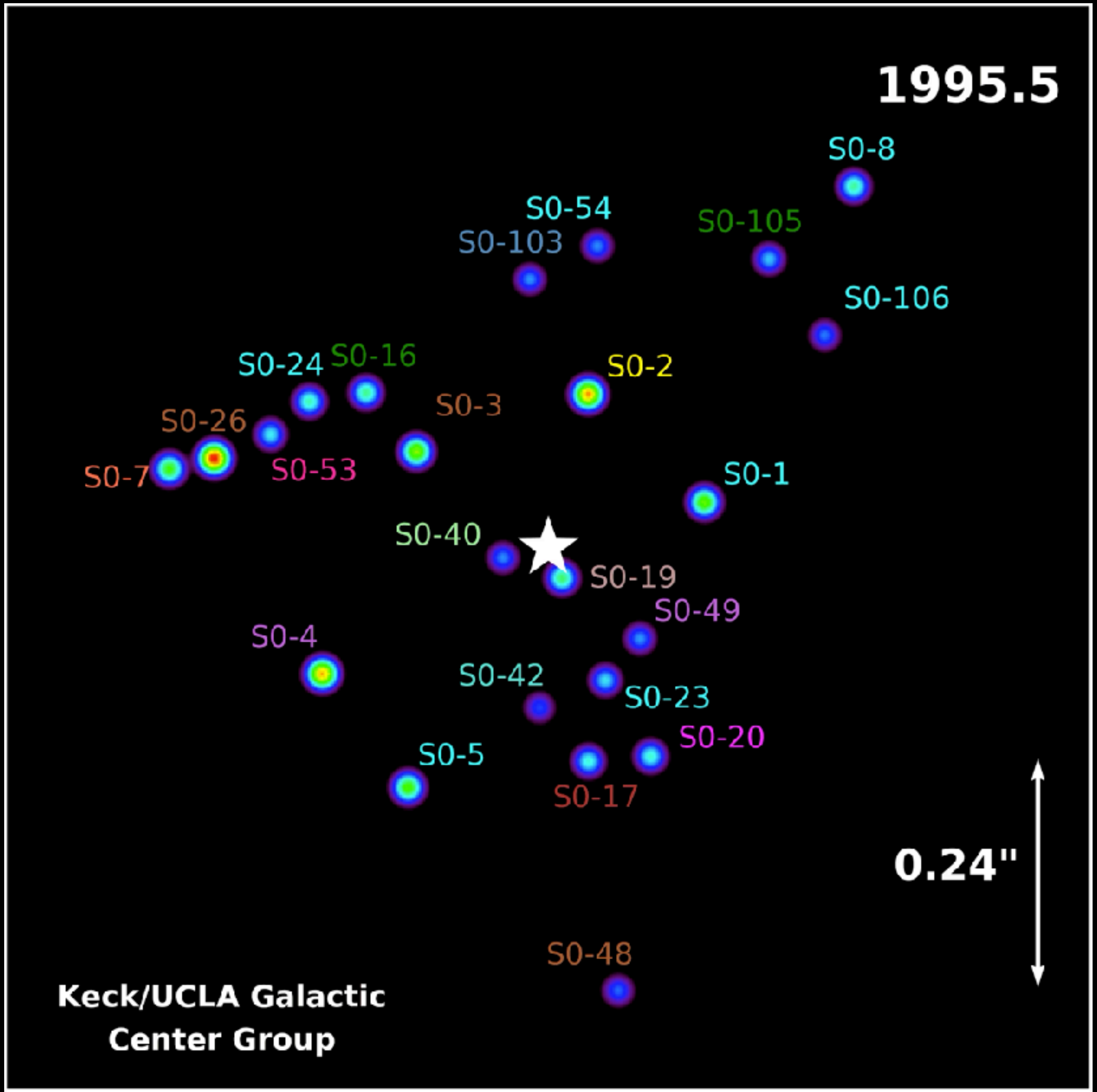


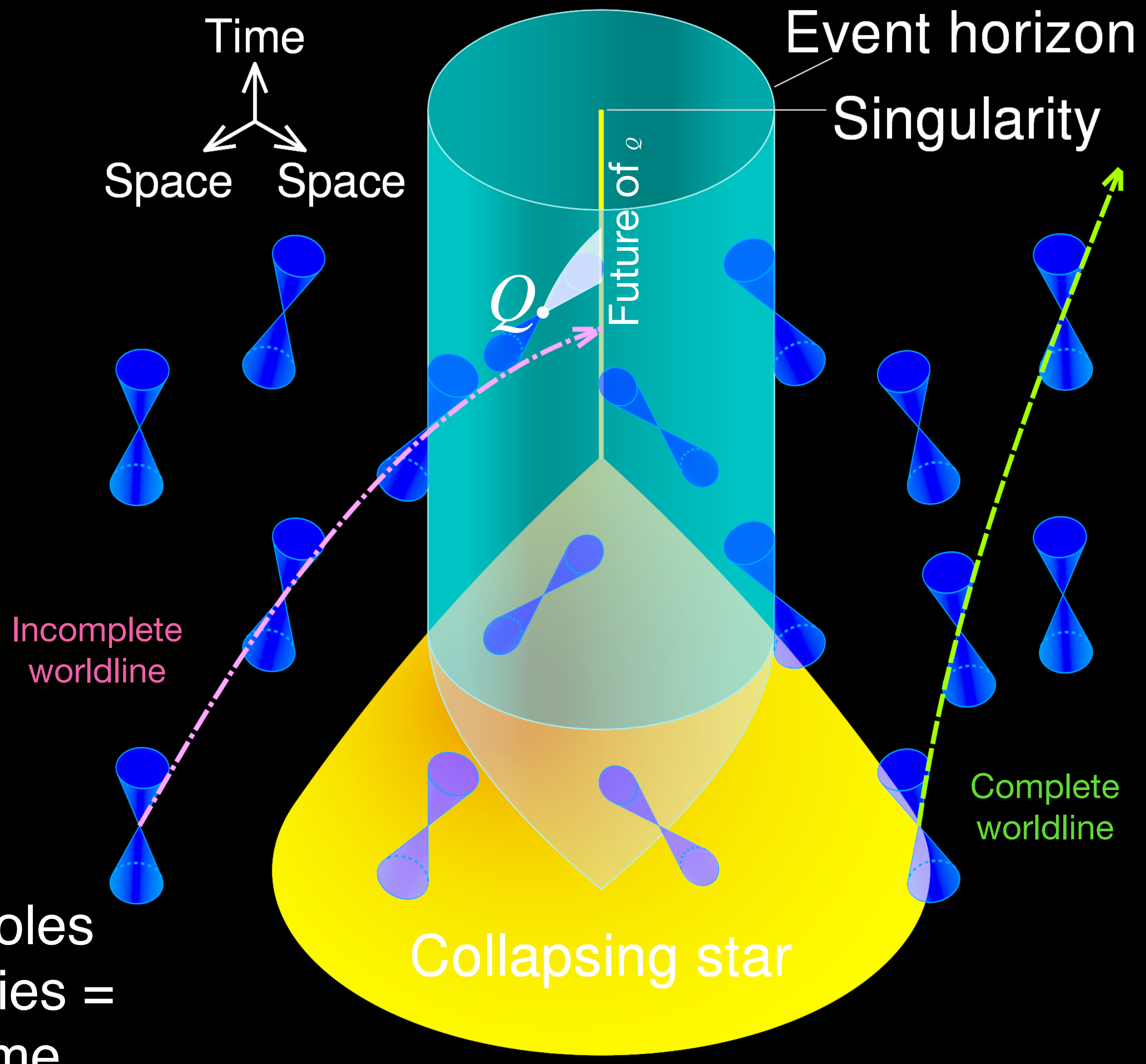
Image & movie courtesy Disney / *The Mandalorian*


Car's future
Events car can reach
moving slower than light
"Time"
Scaled so light travels
on 45° lines
Light's worldline
Car's worldline
"Space"
"Nutwood Ave."
CALIFORNIA
57
50TH ANNIVERSARY
Habit
Photo courtesy Mariah Sanchez , Daily Titan; images courtesy google, wikipedia

2020 Nobel Prize in Physics

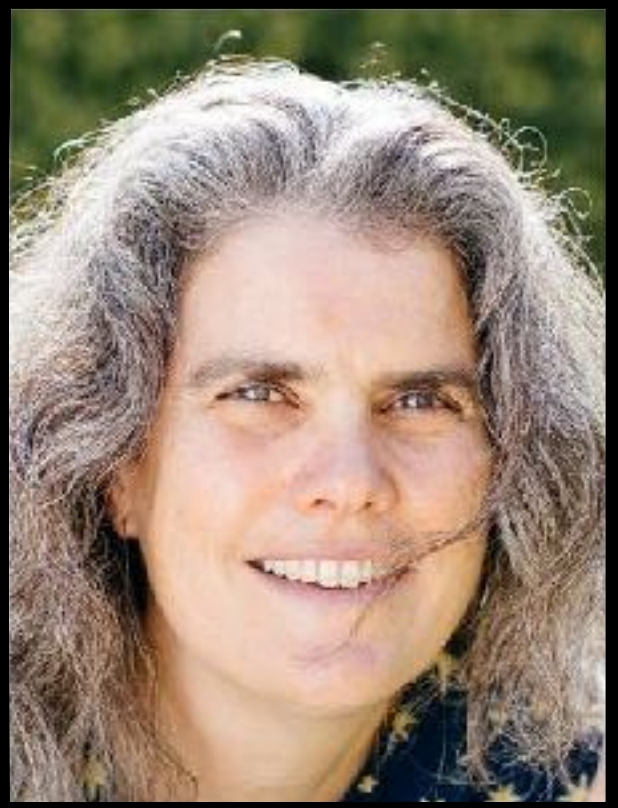


Genzel & Ghez (local at UCLA): there's a black hole at the center of our galaxy



Penrose: black holes contain singularities = edges of spacetime

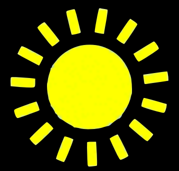
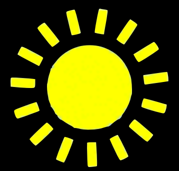
Reinhard Genzel



Andrea Ghez

Roger Penrose

How big are black holes?

- Mass: huge!
  - Two kinds
  - 3 to ~100 
  - Millions+ 
- Radius: small!

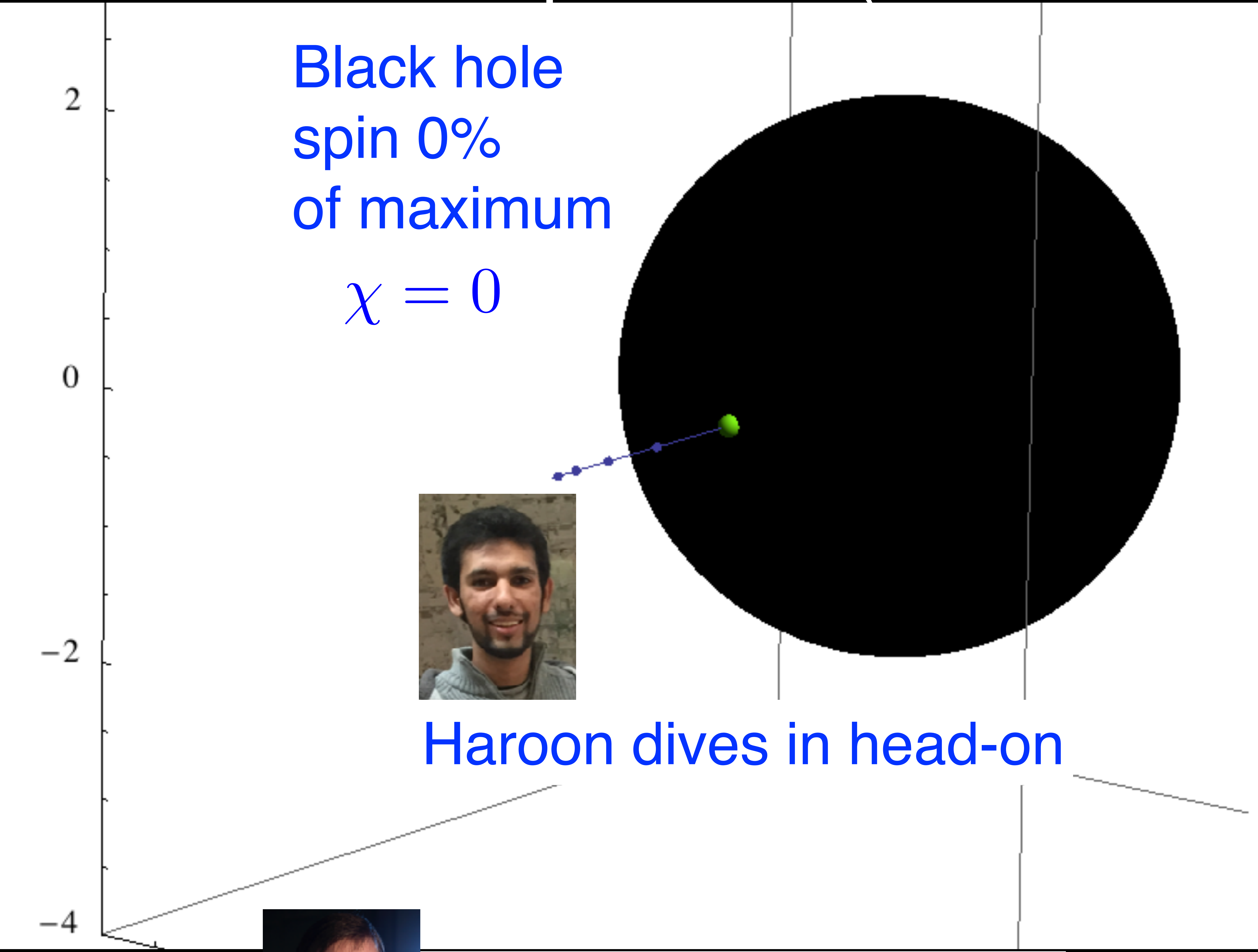
Size of earth-mass black hole

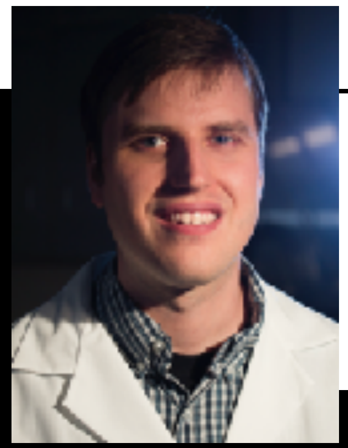
Size of 10-☀ black hole

Map courtesy
Apple maps

Black holes rotate and warp time

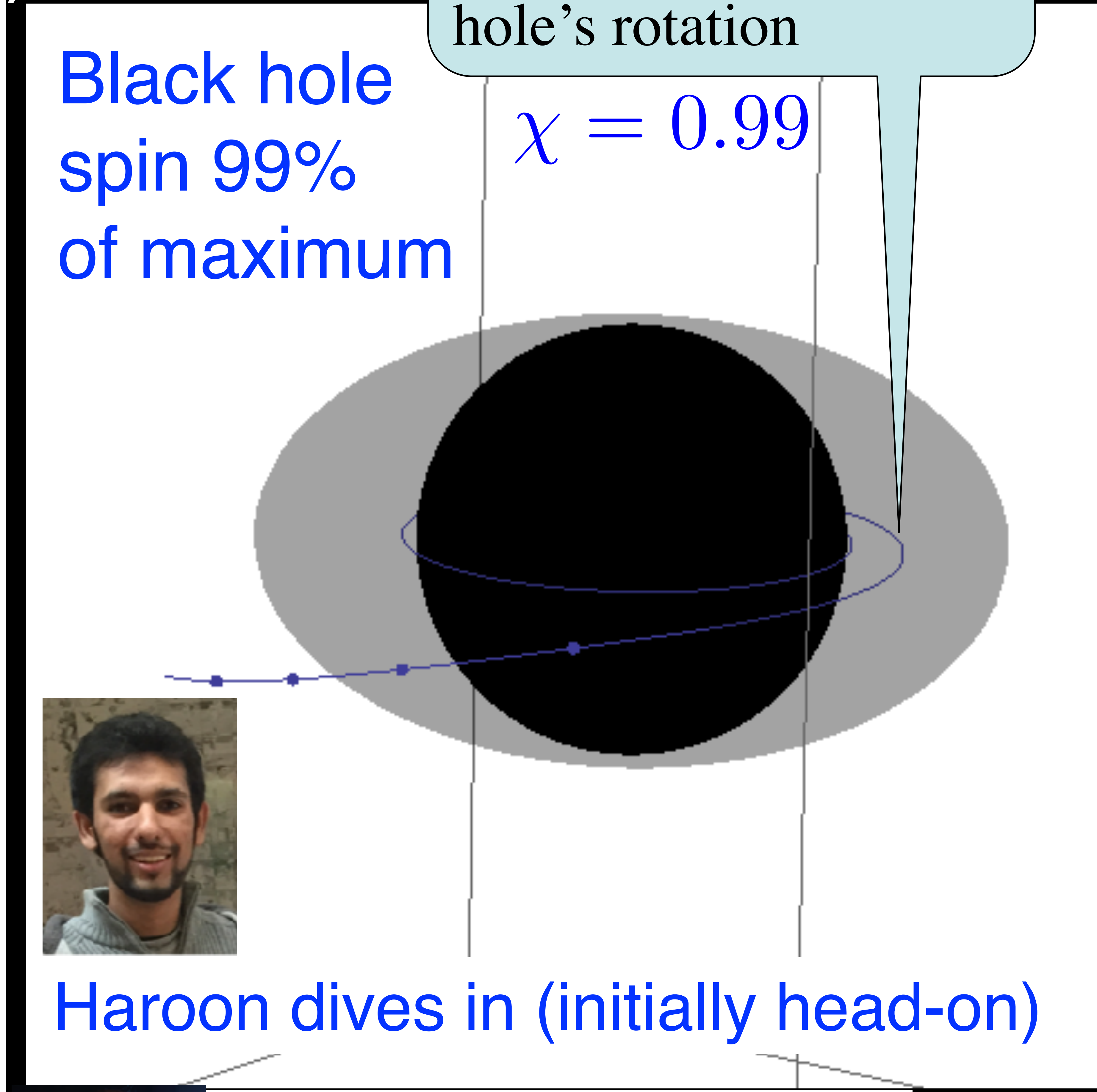
- An experiment (not to scale)

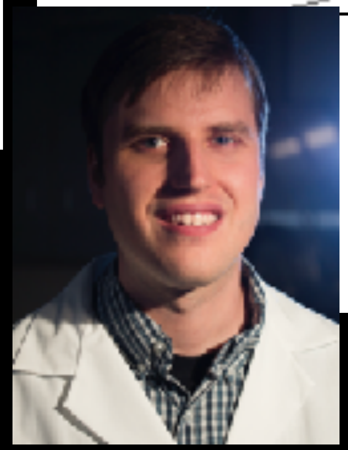




Geoffrey observes from a safe distance

In gray region ("ergosphere"), impossible to avoid rotating around with hole's rotation

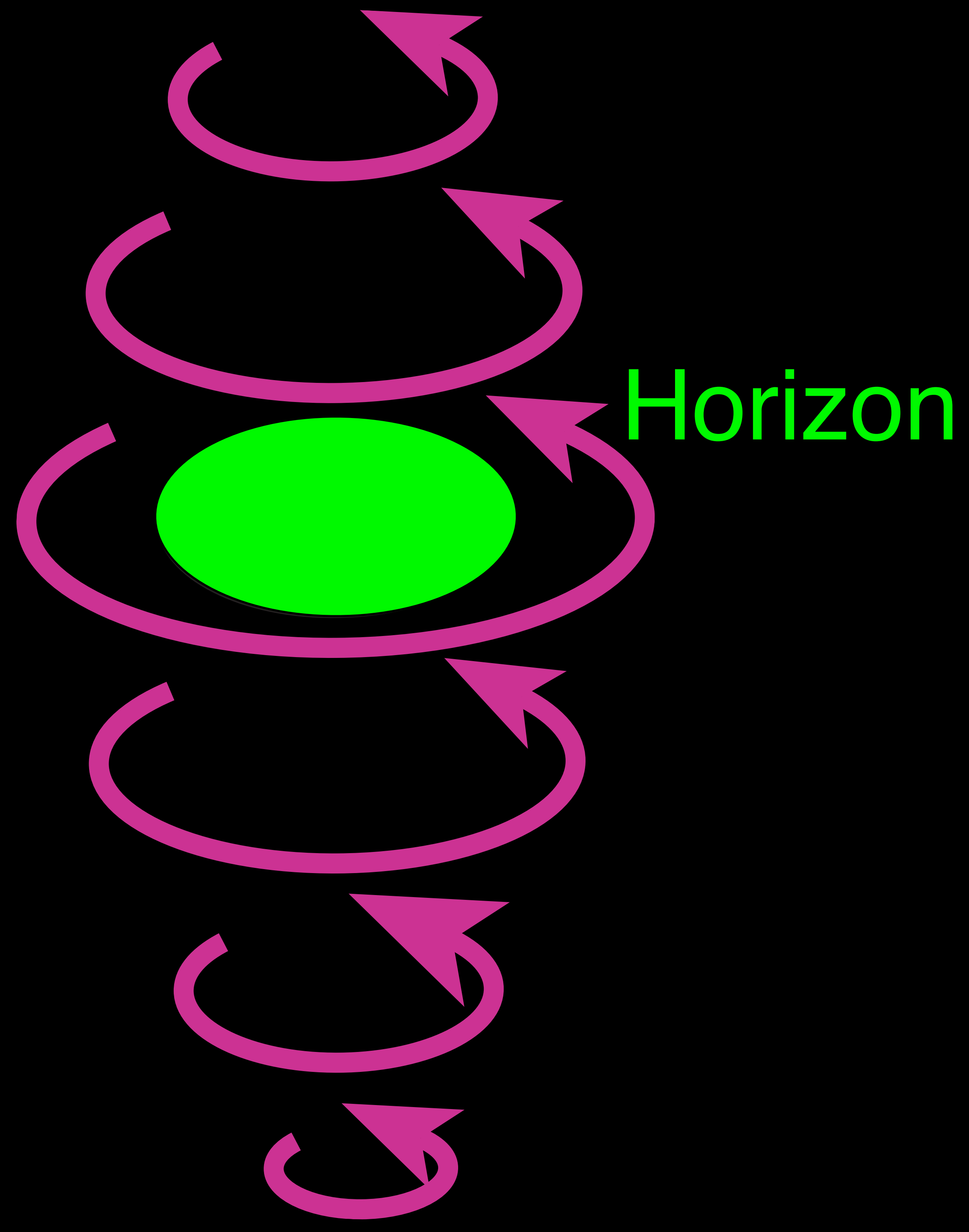


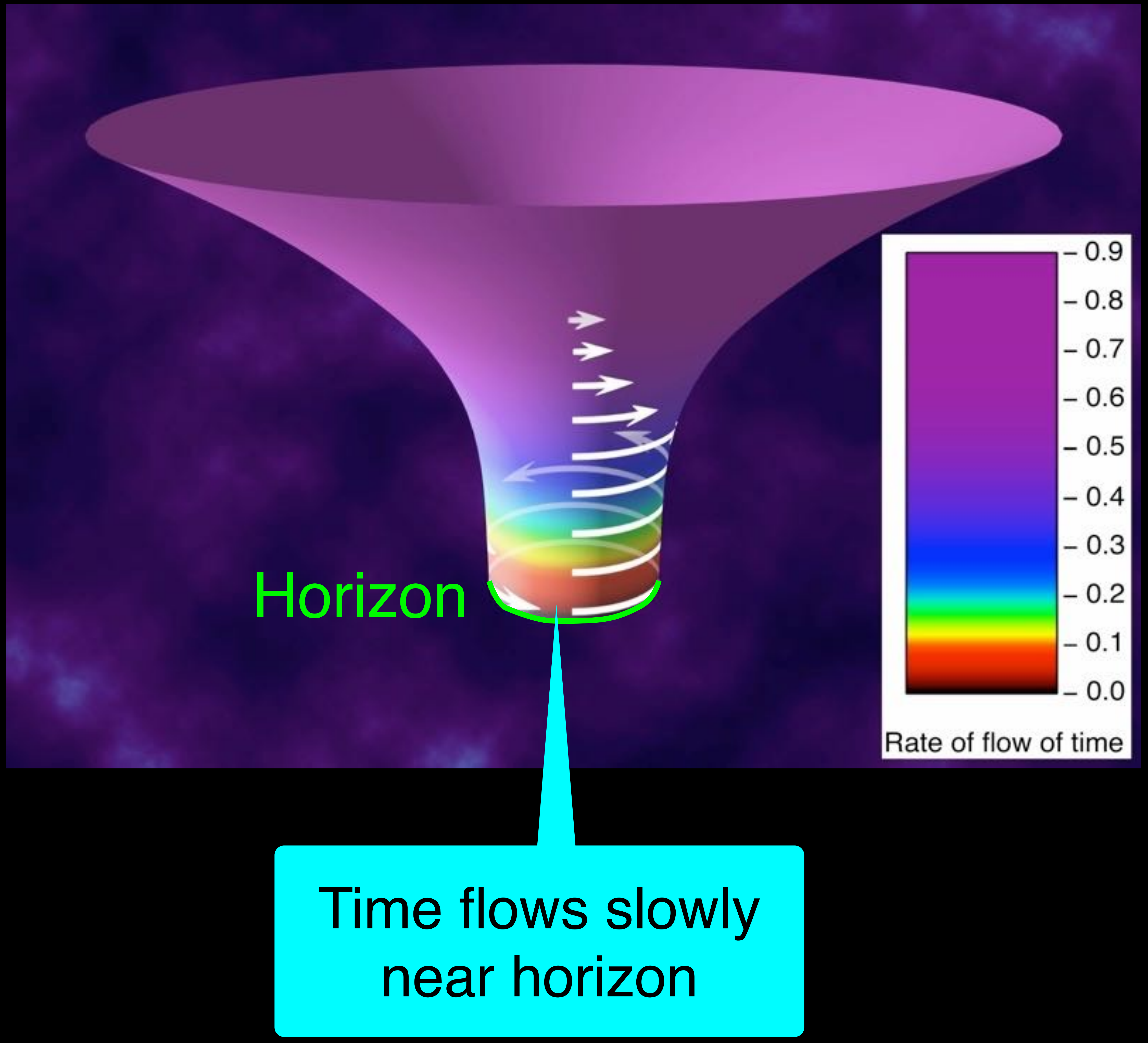


Geoffrey still observes from a safe distance

Black holes rotate and warp time

- Whirl space like a tornado





Images courtesy Kip Thorne

Extremely curved spacetime: black holes

- Gravity so strong…
  - Nothing (even light) can escape from inside hole's horizon (surface)
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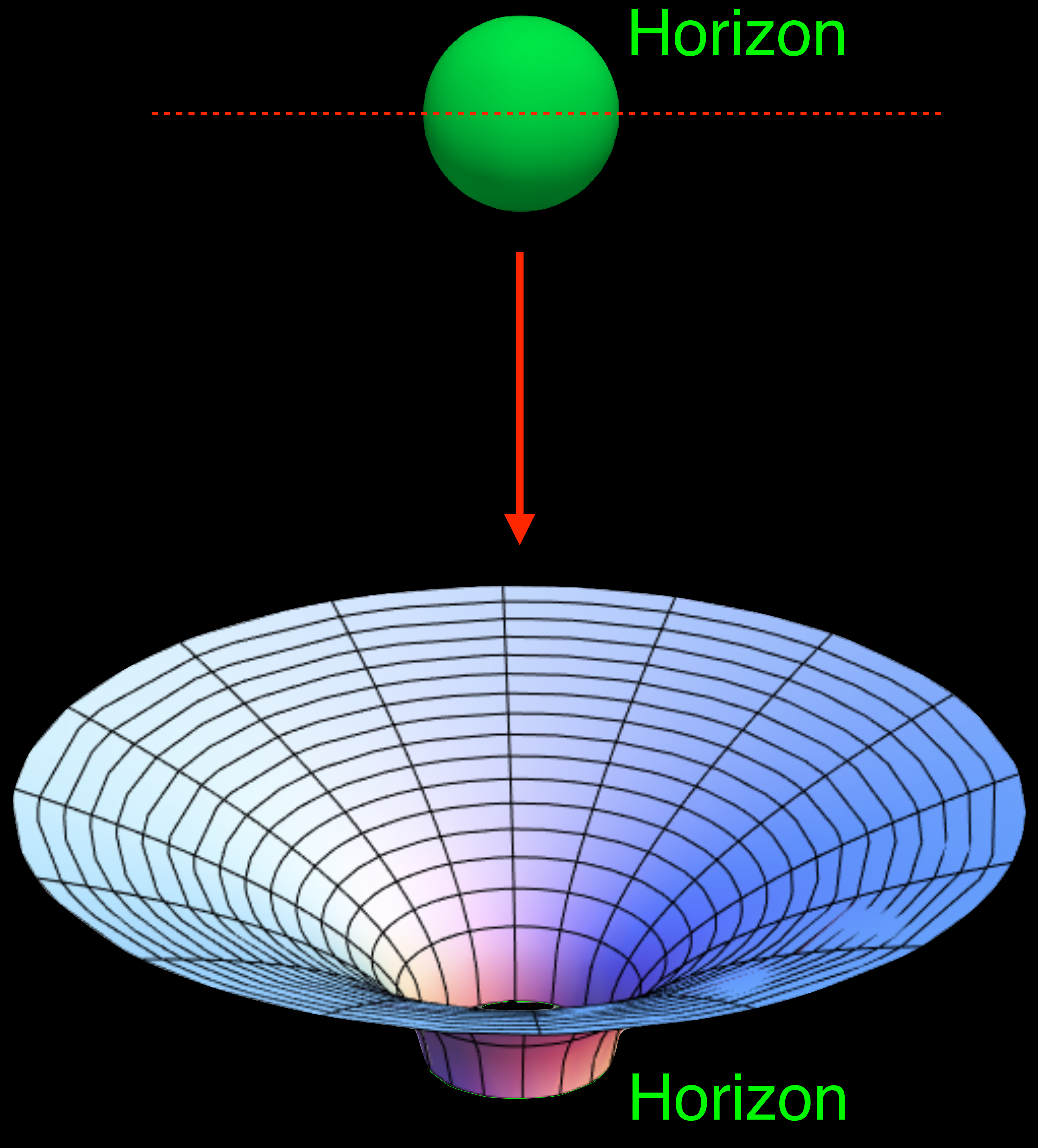
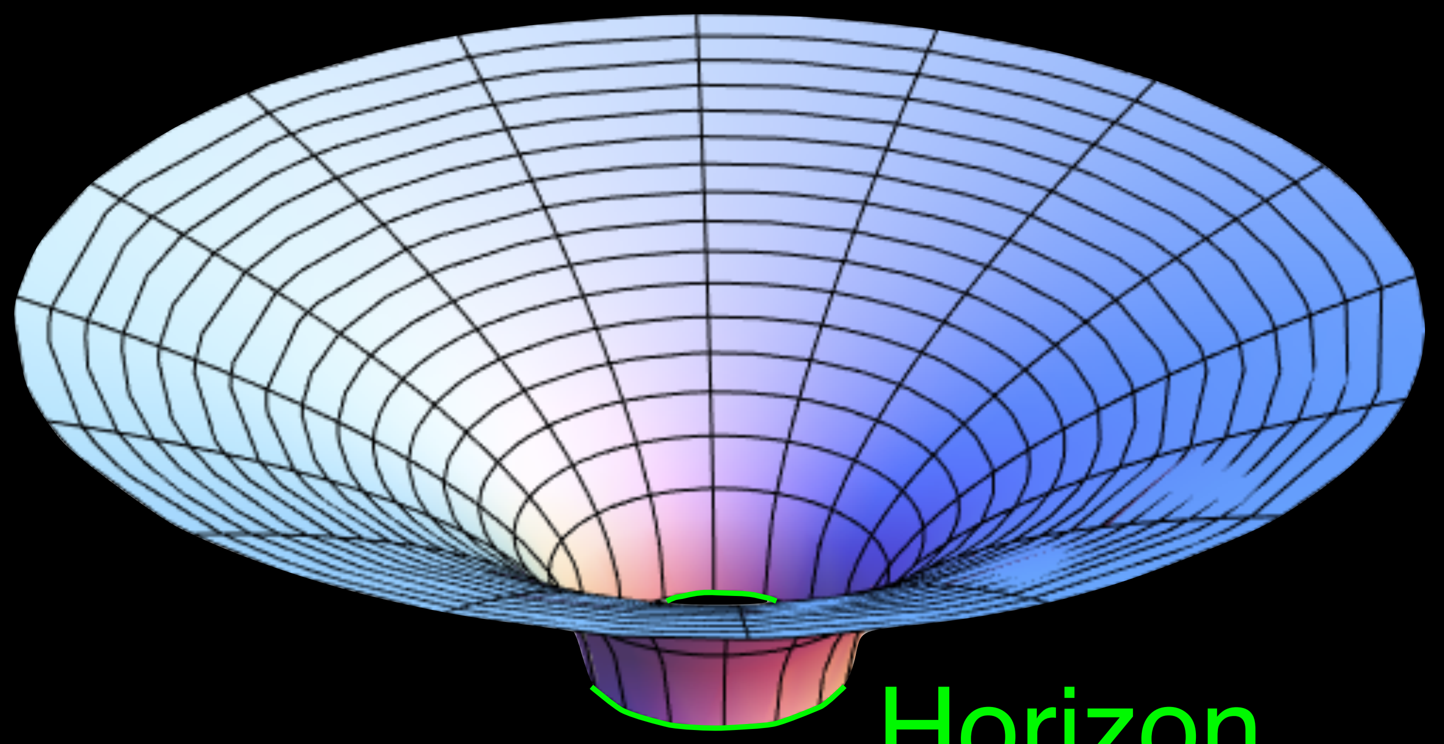
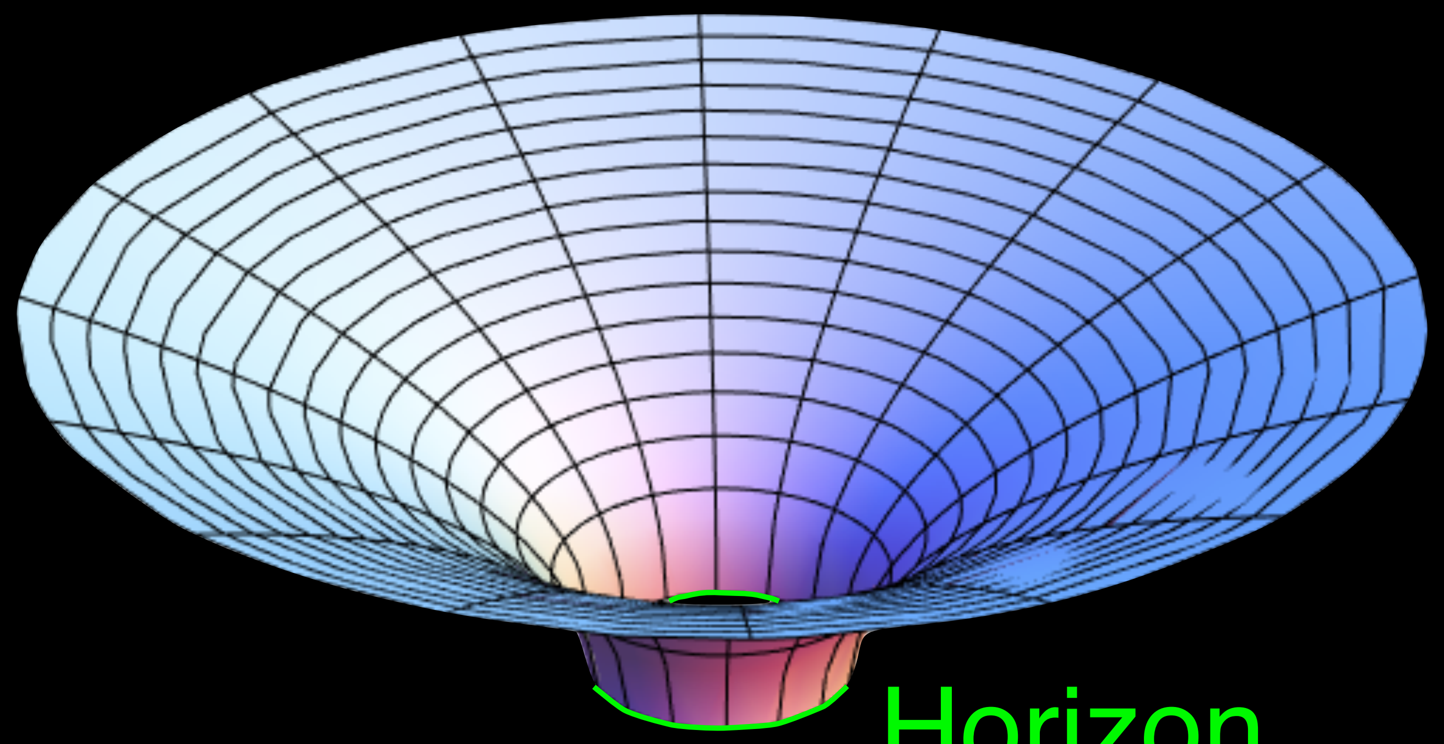



Horizon

+


Horizon

= ?

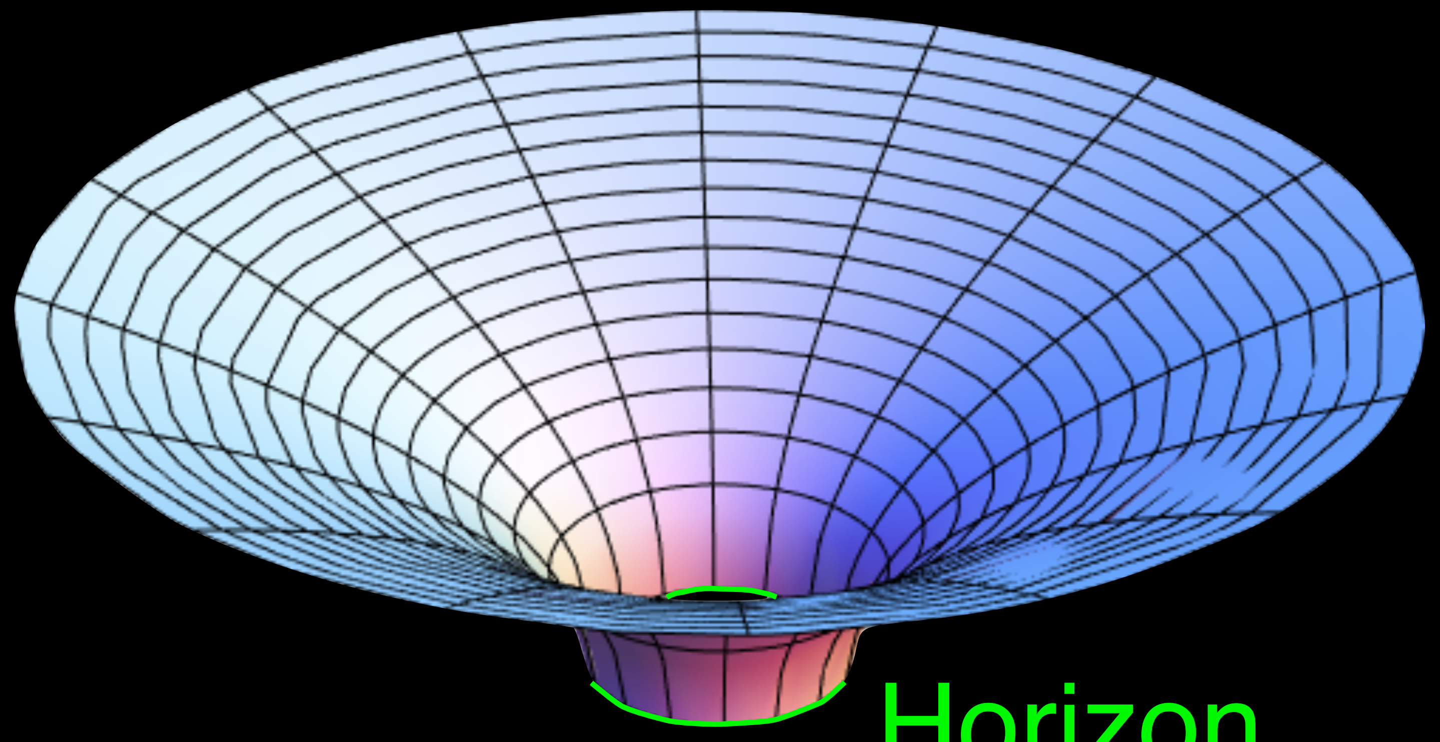
Linear and nonlinear physics

- Linear
  - Whole is sum of parts
  - Example: sound in this room
  - Total sound = sum of individual sounds
- Nonlinear
  - Whole is more than sum of parts
  - Example: water + wind
  - Example: two black holes
  - Need supercomputers to predict

Single black hole

Colliding black holes

Images courtesy Kip Thorne



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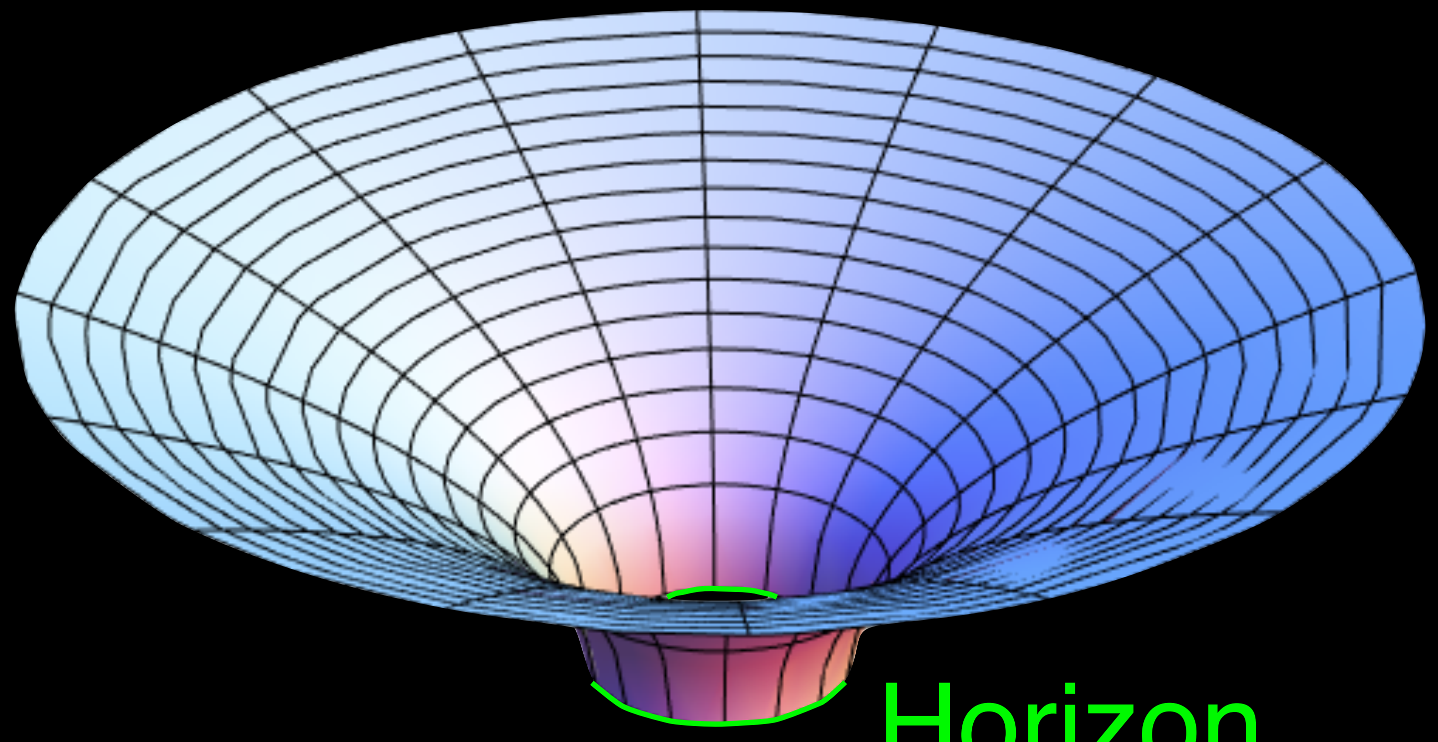

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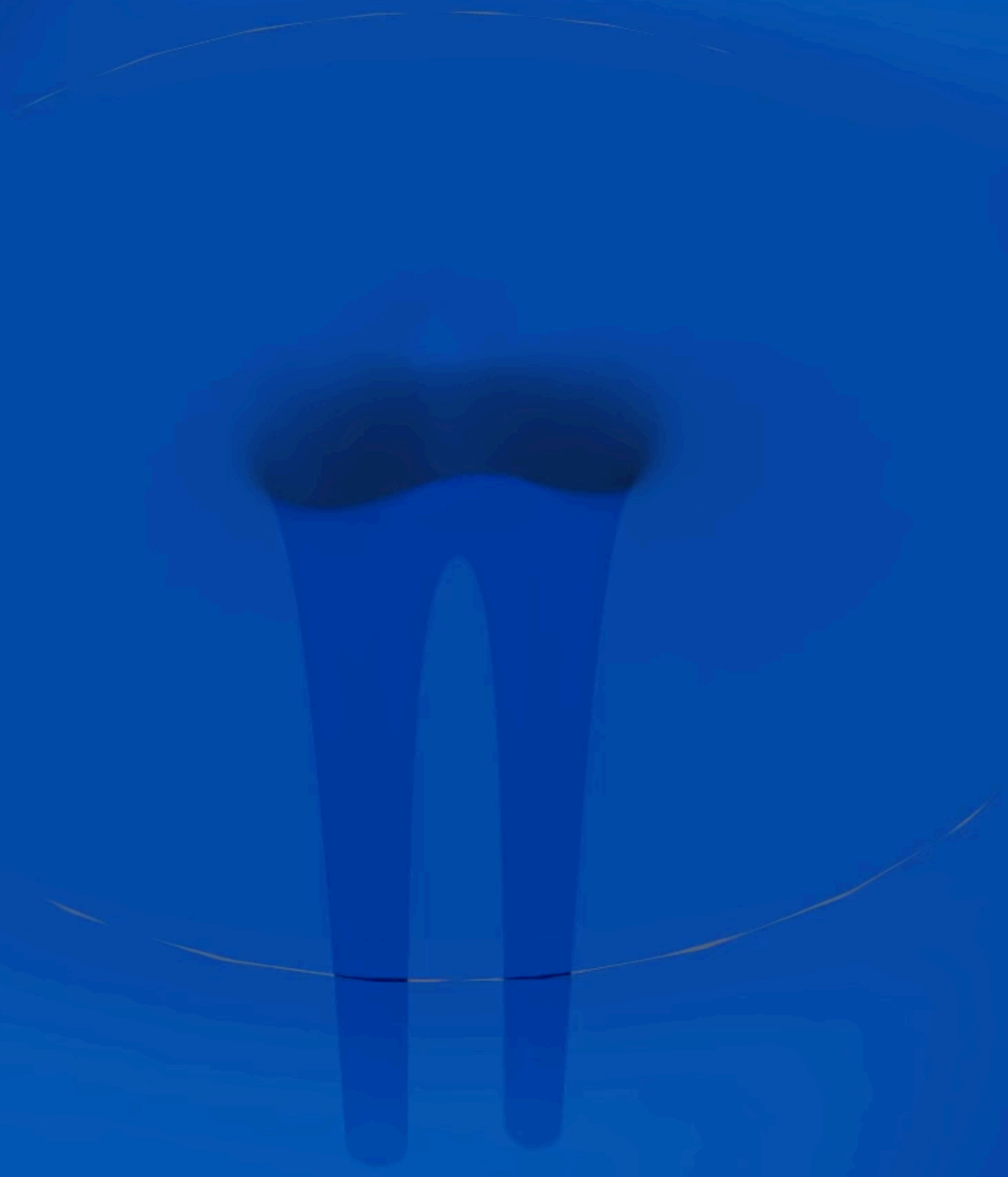
Merging black holes & gravitational waves



By CSUF Undergrad
Nick Demos
(now MIT PhD student)

SXS Collaboration: “Calculation of warped spacetime consistent with GW170104 (zoomed)”

3+1 decomposition

Split spacetime into set of spaces

Spacetime metric = measure of distance between events in spacetime

Goal: evolve (constraint-satisfying) spacetime metric g_{ab}

Spacetime metric $g_{ab}(t=0)$

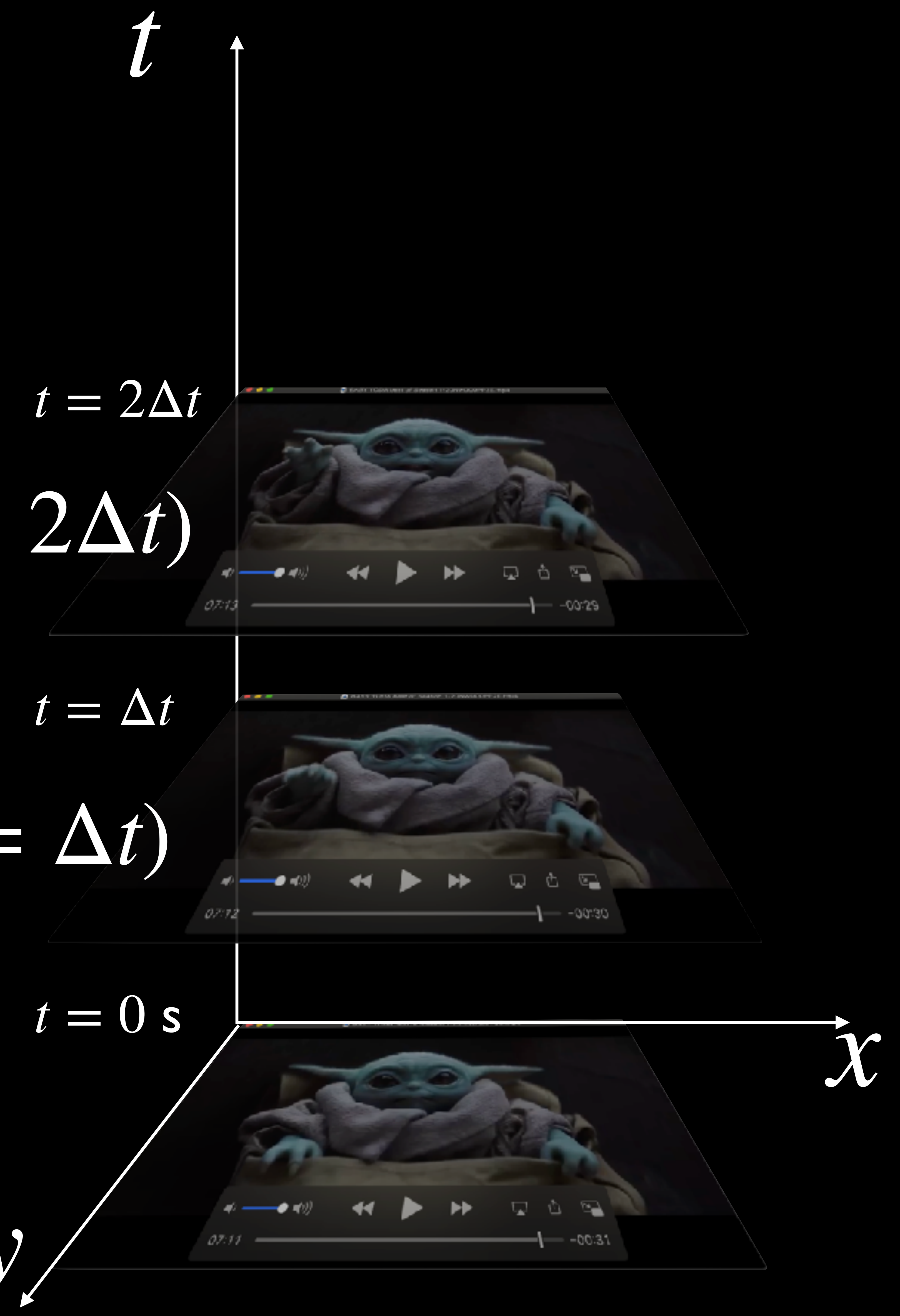
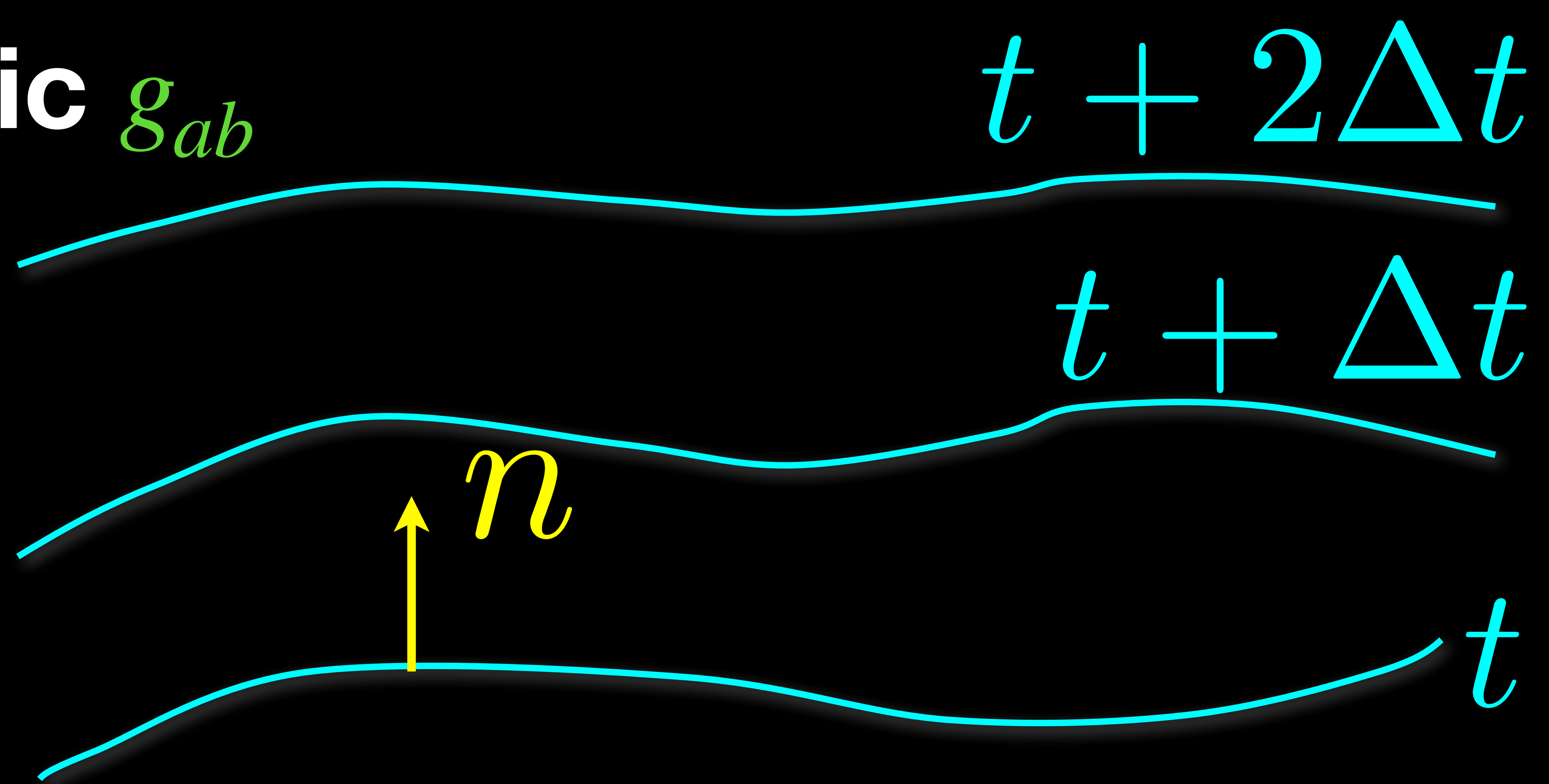


Image & movie courtesy Disney / *The Mandalorian*

Solving Einstein's equations in vacuum

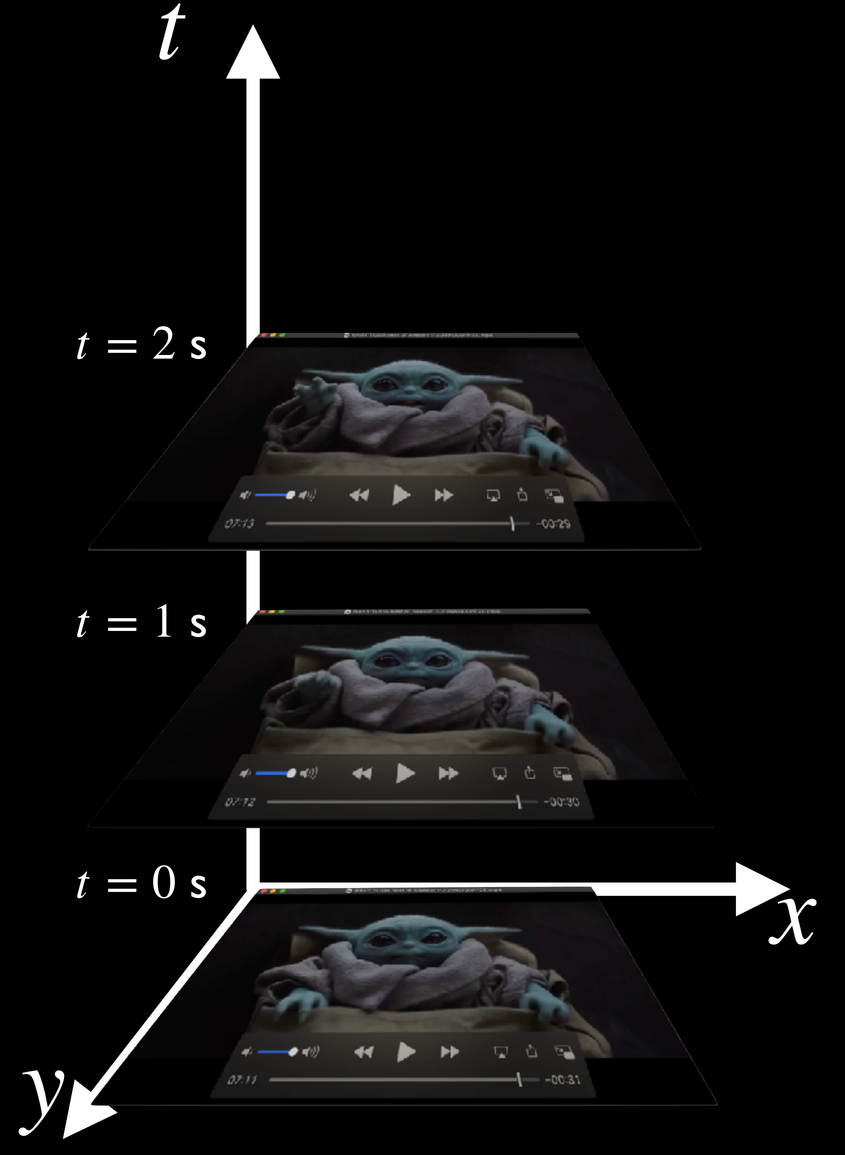
Goal: solve $G_{\mu\nu} = 0$ **for spacetime metric** g_{ab}



- Split spacetime into space + time
  - Constraint equations $G_{nj} = 0 \quad G_{nn} = 0$
    - Solve to create initial data $\text{like } \nabla \cdot E = 0, \nabla \cdot B = 0$
  - Evolution equations $G_{ij} = 0$
    - Constraints must stay satisfied

$$\text{like } \frac{\partial B}{\partial t} = -\nabla \times E,$$
$$\frac{\partial E}{\partial t} = \nabla \times B$$

    - Step 1: Step forward in time
    - Step 2: Repeat step 1 (a lot)



The actual equations we solve

$$
\begin{aligned}
\partial_t g_{ab} - (1+\gamma_1)\beta^k \partial_k g_{ab} &= -\alpha \Pi_{ab} - \gamma_1 \beta^i \Phi_{iab}, \\
\partial_t \Pi_{ab} - \beta^k \partial_k \Pi_{ab} + \alpha \gamma^{ki} \partial_k \Phi_{iab} - \gamma_1 \gamma_2 \beta^k \partial_k g_{ab} \\
= 2\alpha g^{cd}\left(\gamma^{ij}\Phi_{ica}\Phi_{jdb} - \Pi_{ca}\Pi_{db} - g^{ef}\Gamma_{ace}\Gamma_{bdf}\right) \\
- 2\alpha \nabla_{(a}H_{b)} - \frac{1}{2}\alpha n^c n^d \Pi_{cd}\Pi_{ab} - \alpha n^c \Pi_{ci}\gamma^{ij}\Phi_{jab} \\
+ \alpha\gamma_0\left(2\delta^c{}_{(a}n_{b)} - (1+\gamma_3) g_{ab} n^c\right)\mathcal{C}_c \\
+ 2\gamma_4 \alpha \Pi_{ab} n^c \mathcal{C}_c \\
- \gamma_5 \alpha n^c \mathcal{C}_c \left(\frac{\mathcal{C}_a\mathcal{C}_b - \frac{1}{2} g_{ab}\mathcal{C}_d\mathcal{C}^d}{\epsilon_5 + 2n^d\mathcal{C}_d n^e\mathcal{C}_e + \mathcal{C}_d\mathcal{C}^d}\right) \\
- \gamma_1\gamma_2\beta^i\Phi_{iab} \\
- 16\pi\alpha\left(T_{ab} - \frac{1}{2} g_{ab} T^c{}_c\right), \\
\partial_t \Phi_{iab} - \beta^k \partial_k \Phi_{iab} + \alpha \partial_i \Pi_{ab} - \alpha\gamma_2 \partial_i g_{ab} \\
= \frac{1}{2}\alpha n^c n^d \Phi_{icd}\Pi_{ab} + \alpha\gamma^{jk} n^c \Phi_{ijc}\Phi_{kab} \\
- \alpha\gamma_2 \Phi_{iab},
\end{aligned}
$$

Evolution equations $u_\alpha = \{g_{ab}, \Pi_{ab}, \Phi_{iab}\}$

$$\partial_t u_\alpha + \partial_i F^i_\alpha + B^i_{\alpha\beta}\partial_i u_\beta - S_\alpha = 0.$$

$$C_a = H_a + g^{ij}\Phi_{ija} + t^b\Pi_{ba} - \frac{1}{2} g^i_a \psi^{bc}\Phi_{ibc} - \frac{1}{2} t_a \psi^{bc}\Pi_{bc}$$

$$
\begin{aligned}
C_{ia} \equiv\ & g^{jk}\partial_j\Phi_{ika} - \frac{1}{2} g^j_a \psi^{cd}\partial_j\Phi_{icd} + t^b\partial_i\Pi_{ba} - \frac{1}{2} t_a\psi^{cd}\partial_i\Pi_{cd} \\
& + \partial_i H_a + \frac{1}{2} g^j_a \Phi_{jcd}\Phi_{ief}\psi^{ce}\psi^{df} + \frac{1}{2} g^{jk}\Phi_{jcd}\Phi_{ike}\psi^{cd}t^e t_a \\
& - g^{jk}g^{mn}\Phi_{jma}\Phi_{ikn} + \frac{1}{2}\Phi_{icd}\Pi_{be}t_a\left(\psi^{cb}\psi^{de} + \frac{1}{2}\psi^{be}t^c t^d\right) \\
& - \Phi_{icd}\Pi_{ba}t^c\left(\psi^{bd} + \frac{1}{2} t^b t^d\right) + \frac{1}{2}\gamma_2\left(t_a\psi^{cd} - 2\delta^c_a t^d\right)C_{icd}
\end{aligned}
$$

$$C_{iab} = \partial_i g_{ab} - \Phi_{iab}$$

$$C_{ijab} = 2\partial_{[i}\Phi_{j]ab}$$

$$
\begin{aligned}
\mathcal{F}_a \equiv\ & \frac{1}{2} g^i_a \psi^{bc}\partial_i\Pi_{bc} - g^{ij}\partial_i\Pi_{ja} - g^{ij}t^b\partial_i\Phi_{jba} + \frac{1}{2} t_a\psi^{bc}g^{ij}\partial_i\Phi_{jbc} \\
& + t_a g^{ij}\partial_i H_j + g^i_a \Phi_{ijb}g^{jk}\Phi_{kcd}\psi^{bd}t^c - \frac{1}{2} g^i_a \Phi_{ijb}g^{jk}\Phi_{kcd}\psi^{cd}t^b \\
& - g^i_a t^b\partial_i H_b + g^{ij}\Phi_{icd}\Phi_{jba}\psi^{bc}t^d - \frac{1}{2} t_a g^{ij}g^{mn}\Phi_{imc}\Phi_{njd}\psi^{cd} \\
& - \frac{1}{4} t_a g^{ij}\Phi_{icd}\Phi_{jbe}\psi^{cb}\psi^{de} + \frac{1}{4} t_a\Pi_{cd}\Pi_{be}\psi^{cb}\psi^{de} - g^{ij}H_i\Pi_{ja} \\
& - t^b g^{ij}\Pi_{bi}\Pi_{ja} - \frac{1}{4} g^i_a \Phi_{icd}t^c t^d\Pi_{be}\psi^{be} + \frac{1}{2} t_a\Pi_{cd}\Pi_{be}\psi^{ce}t^d t^b \\
& + g^i_a \Phi_{icd}\Pi_{be}t^c t^b\psi^{de} - g^{ij}\Phi_{iba}t^b\Pi_{je}t^e - \frac{1}{2} g^{ij}\Phi_{icd}t^c t^d\Pi_{ja} \\
& - g^{ij}H_i\Phi_{jba}t^b + g^i_a\Phi_{icd}H_b\psi^{bc}t^d + \gamma_2\left(g^{id}\mathcal{C}_{ida} - \frac{1}{2} g^i_a\psi^{cd}\mathcal{C}_{icd}\right) \\
& + \frac{1}{2} t_a\Pi_{cd}\psi^{cd}H_b t^b - t_a g^{ij}\Phi_{ijc}H_d\psi^{cd} + \frac{1}{2} t_a g^{ij}H_i\Phi_{jcd}\psi^{cd} \\
& - 16\pi t^a T_{ab}
\end{aligned}
$$

Constraint equations

$$H_a \equiv g_{ab}\partial^c\partial_c x^b$$

$a, b, \ldots =$ spacetime indices t, x, y, z

$i, j, \ldots =$ spatial indices x, y, z

$\alpha, \beta, \ldots =$ equation indices $g_{ab}, \Pi_{ab}, \Phi_{iab}$

Sum over repeated indices

$$G = c = 1$$

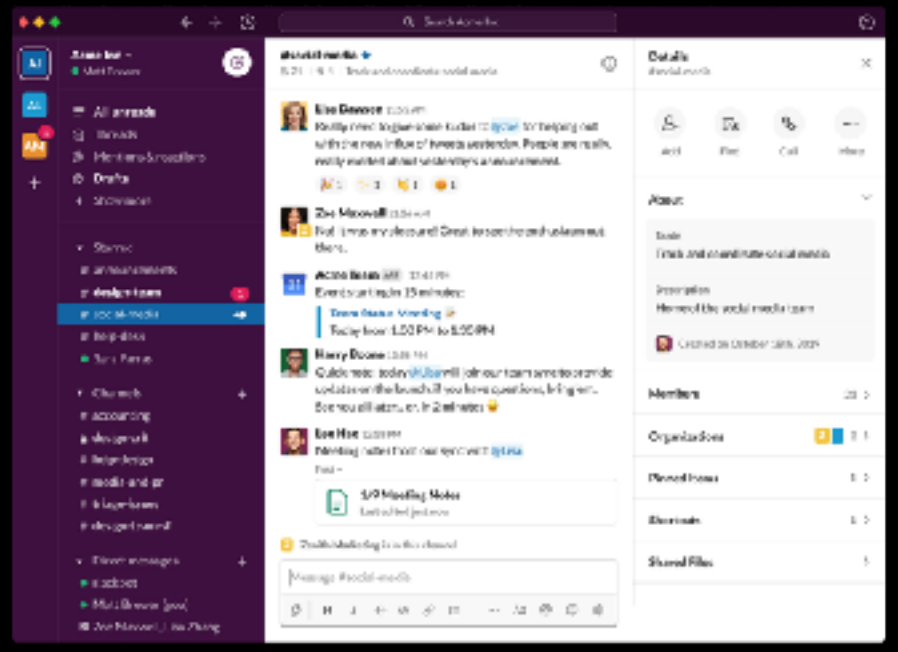
Lindblom+ Class. Quant. Grav. **23**, S447 (2006)

SpECTRE

- Open, next-gen. NR code
  - Discontinuous Galerkin (DG)
  - Task-based parallelism

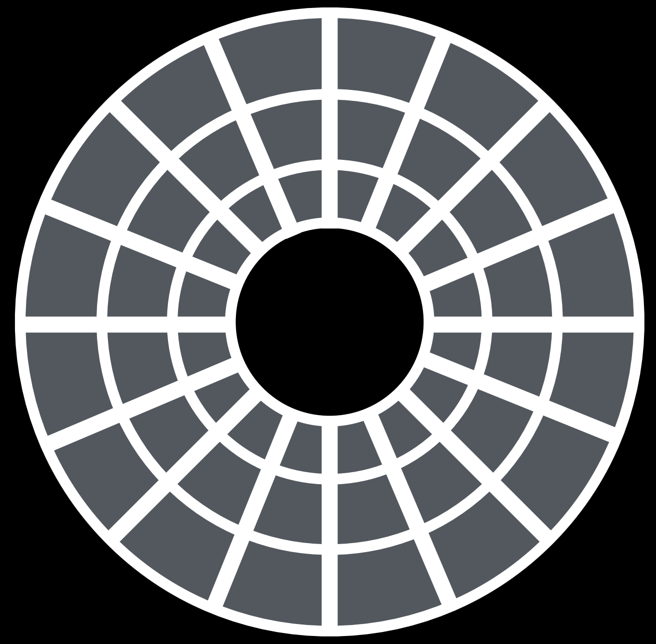
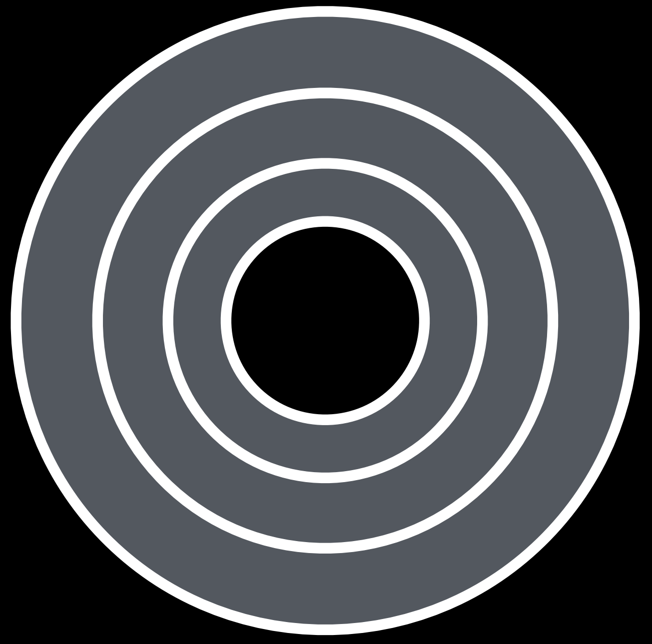
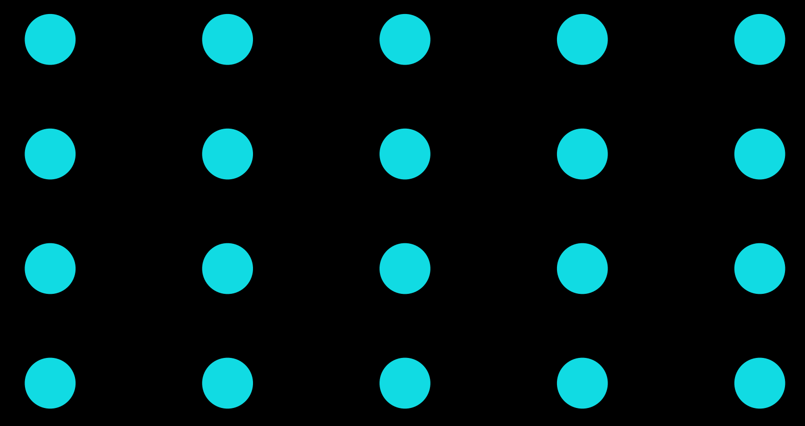
SpEC	SpECTRE
Home-grown	charm++ charm.cs.illinois.edu
Cores run same code on different parts of grid	Cores ask scheduler for tasks from queue

Many sync points



Few sync points

Scales to 50 cores	**Scales to 100k cores**

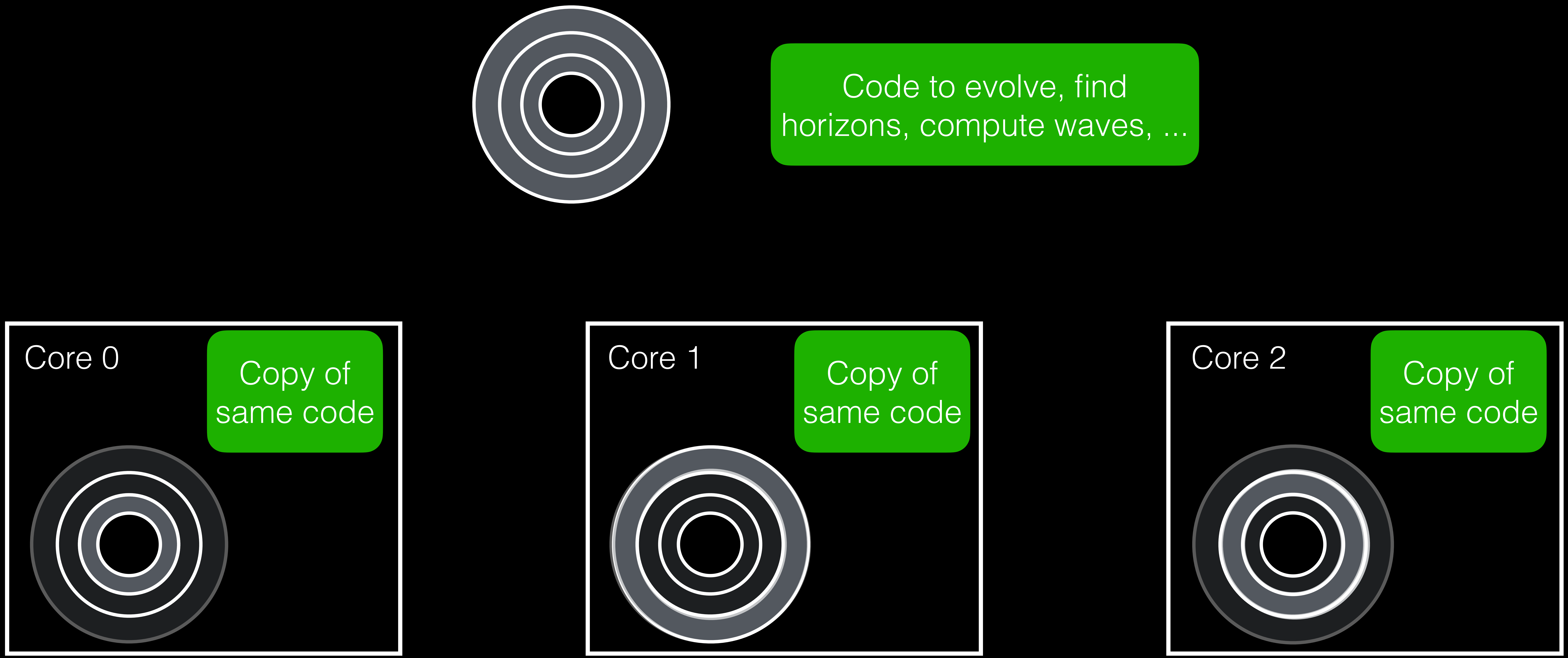
DG	Pseudospectral	Finite Diff.
		 x_{n-1} x_n x_{n+1}

$$f(x) = \sum_{n=0}^{N} a_n \phi(x)$$

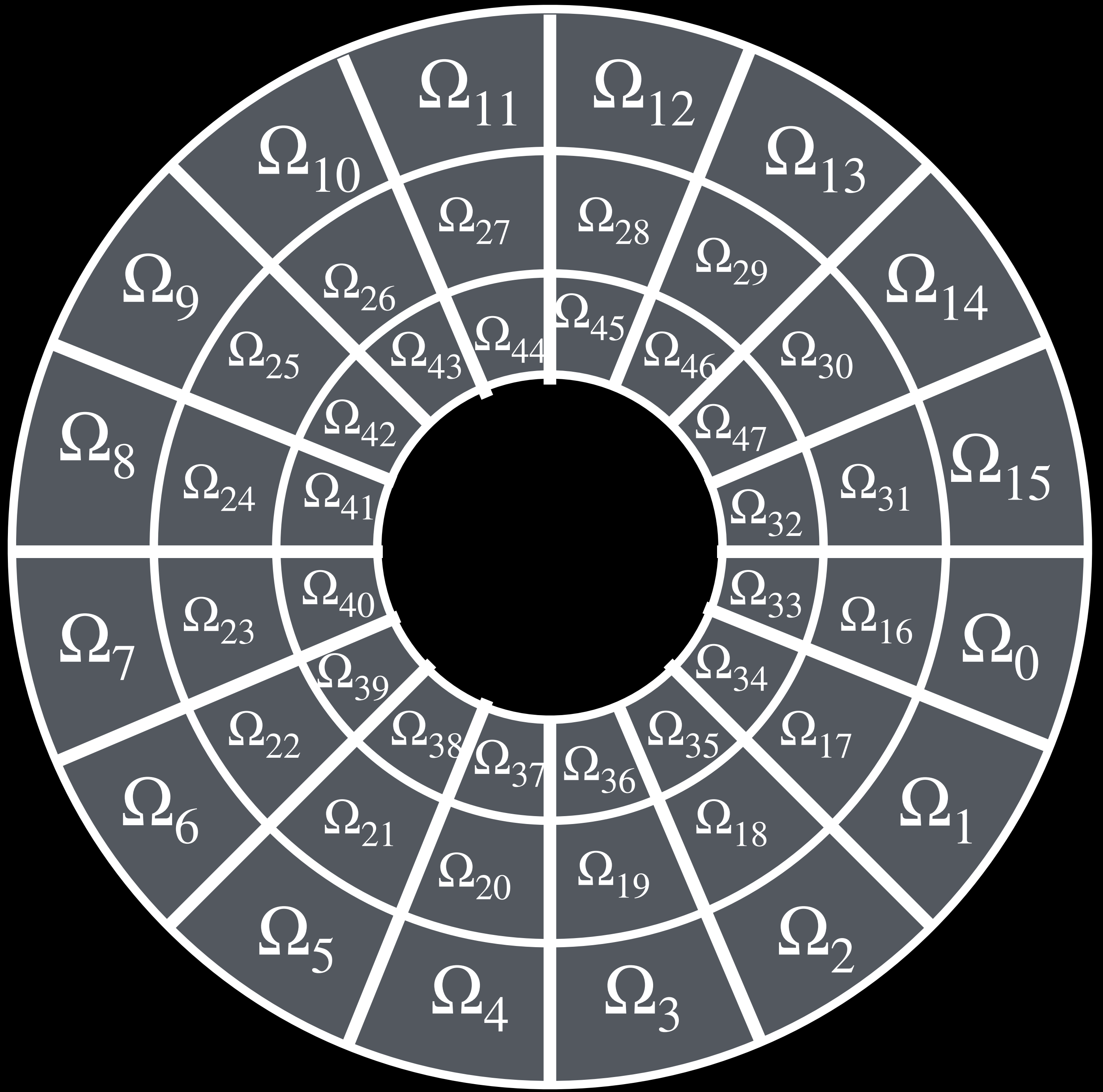
DG	Pseudospectral	Finite Diff.
Smaller N more cells	Bigger N fewer cells	Values at grid points
		Shocks
Exponential convergence *when solution smooth*		Polynomial convergence
Analytic high-order derivatives		Wide stencils *High communication on many CPUs*

Images courtesy zoom.us, slack.com

Data parallelism



Task parallelism



= array parallel component

= singleton parallel component

= global cache

Parallel component

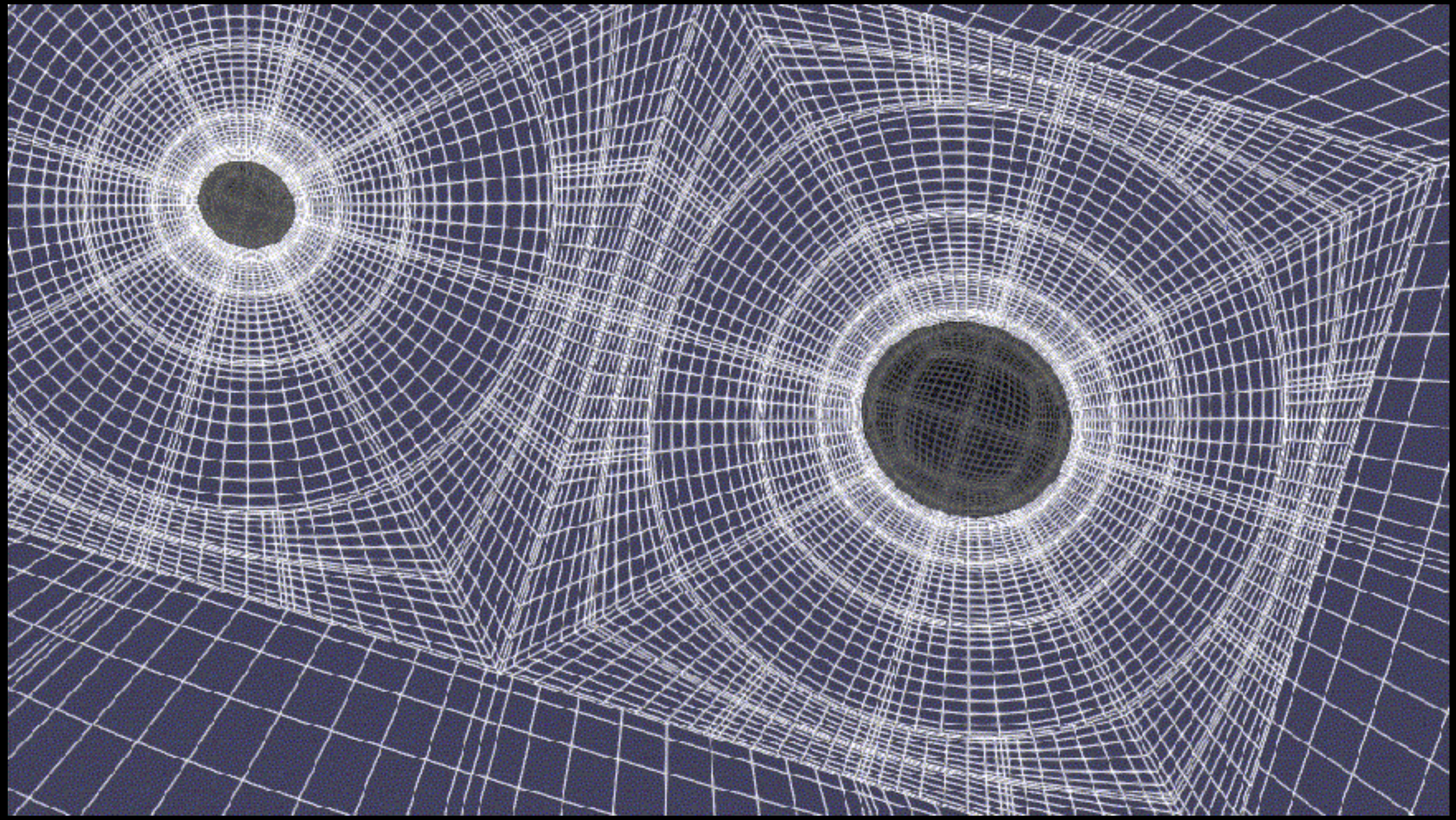
= "actor" that knows things, does things

= "distributed object"

= charm++ chare

Moving mesh

- Deform, move mesh with grid velocity
  - Track black holes, ensuring singularities remain excised, horizon exteriors not excised



Animation courtesy Kyle Nelli